

Mina Talati Vinod Kumar Yadav

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Applied Physics-I

by Mina Talati, Vinod Kumar Yadav **[English Edition]**

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प्रो. अनिल डी. सहस्रबद्धे अध्यक्ष Prof. Anil D. Sahasrabudhe Chairman

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ALL INDIA COUNCIL FOR TECHNICAL EDUCATION (A STATUTORY BODY OF THE GOVT. OF INDIA) (Ministry of Education, Govt. of India) Nelson Mandela Marg, Vasant Kunj, New Delhi-110070
Phone : 011-26131498 E-mail : chairman@aicte-india.org

FOREWORD

Engineering has played a very significant role in the progress and expansion of mankind and society for centuries. Engineering ideas that originated in the Indian subcontinent have had a thoughtful impact on the world.

All India Council for Technical Education (AICTE) had always been at the forefront of assisting Technical students in every possible manner since its inception in 1987. The goal of AICTE has been to promote quality Technical Education and thereby take the industry to a greater heights and ultimately turn our dear motherland India into a Modern Developed Nation. It will not be inept to mention here that Engineers are the backbone of the modern society - better the engineers, better the industry, and better the industry, better the country.

NEP 2020 envisages education in regional languages to all, thereby ensuring that each and every student becomes capable and competent enough and is in a position to contribute towards the national growth and development.

One of the spheres where AICTE had been relentlessly working from last few years was to provide high-quality moderately priced books of International standard prepared in various regional languages to all it's Engineering students. These books are not only prepared keeping in mind it's easy language, real life examples, rich contents and but also the industry needs in this everyday changing world. These books are as per AICTE Model Curriculum of Engineering & Technology - 2018.

Eminent Professors from all over India with great knowledge and experience have written these books for the benefit of academic fraternity. AICTE is confident that these books with their rich contents will help technical students master the subjects with greater ease and quality.

AICTE appreciates the hard work of the original authors, coordinators and the translators for their endeavour in making these Engineering subjects more lucid.

- Realmy

(Anil D. Sahasrabudhe)

ACKNOWLEDGEMENT

The author(s) are grateful to AICTE for their meticulous planning and execution to publish the technical book for Diploma students.

We sincerely acknowledge the valuable contributions of the reviewer of the book Prof. Medha Shirish Gijare, for making it students' friendly and giving a better shape in an artistic manner.

This book is an outcome of various suggestions of AICTE members, experts and authors who shared their opinion and thoughts to further develop the engineering education in our country.

It is also with great honour that we state that this book is aligned to the AICTE Model Curriculum and in line with the guidelines of National Education Policy (NEP) -2020. Towards promoting education in regional languages, this book is being translated in scheduled Indian regional languages.

Acknowledgements are due to the contributors and different workers in this field whose published books, review articles, papers, photographs, footnotes, references and other valuable information enriched us at the time of writing the book.

Finally, we like to express our sincere thanks to the publishing house, M/s. Khanna Book Publishing Company Private Limited, New Delhi, whose entire team was always ready to cooperate on all the aspects of publishing to make it a wonderful experience.

> **Mina Talati Vinod Kumar Yadav**

Preface

The book titled 'Applied Physics-I' is an outcome of the experience of our teaching of fundamental physics courses at the diploma engineering level. We have included the topics of fundamentals of physics relevant to diploma engineering courses and as per the requirements of Outcome Based Education (OBE) model curriculum of AICTE for 1st-year diploma engineering and new National Education Policy (NEP) 2020. Sincere efforts have been made to keep the content of this book interesting, student-friendly and lucid while at the same time revealing and explaining fundamental concepts of physics to the engineering students have not been compromised.

Throughout the preparation of the manuscript of this book, we have considered various standard textbooks, research papers, and reports, and accordingly, we have included questions, solved and supplementary problems. The book covers problems of different difficulty levels which certainly can be solved with some thoughtful efforts. In this book, the emphasis has been laid on definitions of physical phenomena and physical quantities, laws of physics, and various physics formulae relevant to the curriculum for quick revision of basic principles. We have also tried to provide various illustrations and examples in each unit for a proper understanding of the concepts of physics that a student can relate to. For further clarification of the concepts, we have used the figures and diagrams available under fair use policies and creative commons licenses.

It is important to note that, we have included the relevant twelve laboratory practical as per curriculum at the end of each unit. In addition, we have put together some essential formulae and conversion of units in the annexure section. In each unit, video and/ simulation links have been given to support and boost the user's desire for self-learning of the topics within the limits of the curriculum.

We sincerely hope that the book will create curiosity and inspire the students to learn, discuss and make use of the basic principles of physics for addressing the problems related to their core disciplines. The reader's beneficial comments and suggestions will play a major role in improving the future editions of the book. It gives us immense pleasure to place this book, written under 'Technical Book Writing Scheme' for 1st-year diploma engineering, in the hands of the teachers and students.

> **Mina Talati Vinod Kumar Yadav**

Outcome Based Education

The outcome-based curriculum has been developed for the implementation of an outcome-based education for diploma engineering students. It incorporates the outcome-based assessment also through which educators and evaluators will be able to assess and evaluate the achievement of students in the form of standard, specific and measurable program outcomes. Outcome-based education emphasizes achieving program-specific skills systematically and gradually which diploma engineering students must acquire. Through outcome-based education,learners will be able to commit to achieving a minimum standard without quitting the program at any level. Upon completion of the specific program with an outcome-based education strategy, diploma engineering students will be able to arrive at the following program outcomes:

- **1. Basic and Discipline specific knowledge**: Apply knowledge of basic mathematics, science and engineering fundamentals and engineering specialization to solve the engineering problems.
- **2. Problem analysis:** Identify and analyse well-defined engineering problems using codified standard methods.
- **3. Design/ development of solutions:** Design solutions for well-defined technical problems and assist with the design of systems components or processes to meet specified needs.
- **4. Engineering Tools, Experimentation and Testing**: Apply modern engineering tools and appropriate technique to conduct standard tests and measurements.
- **5. Engineering practices for society, sustainability and environment:** Apply appropriate technology in context of society, sustainability, environment and ethical practices.
- **6. Project Management:** Use engineering management principles individually, as a team member or a leader to manage projects and effectively communicate about well-defined engineering activities.
- **7. Life-long learning**: Ability to analyse individual needs and engage in updating in the context of technological changes.

Course Outcomes

After completion of the course the students will be able to:

- CO-1: Select the physical quantities for accurate and precise measurements of engineering problems and estimate the errors in measurements.
- CO-2: Perform addition, subtraction, multiplication (scalar and vector product) of vector and find resolution of vectors for relevant applications. Analyse and apply the type of motions to resolve the engineering applications.
- CO-3: Define scientific terms work, energy and power and their units and derive relationships between them to solve engineering problems. Describe forms of friction and methods to minimize friction between different surfaces. State the principle of conservation of energy and identify various forms of energy and energy transformations.
- CO-4: Compare and relate physical properties associated with linear motion and rotational motion and apply conservation of angular momentum principle to known problems
- CO-5: Select relevant materials in industry by analysing the physical properties of solids and liquids to solve broad-based engineering problems.
- CO-6: Explain the basic principles of heat and measure the temperature using various thermometers. Identify and apply modes of heat transfer by knowing coefficient of expansion and thermal conductivity of material in related engineering applications.

Abbreviations and Symbols

List of Abbreviations

List of Symbols

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List of figures

*Unit 4***: Rotational Motion**

*Unit 5***: Properties of Matter**

*Unit 6***: Heat and Thermometry**

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Guidelines for Teachers

To implement Outcome Based Education (OBE) knowledge level and skill set of the students should be enhanced. Teachers should take a major responsibility for the proper implementation of OBE. Some of the responsibilities (not limited to) for the teachers in OBE system may be as follows:

- Within reasonable constraints, they should channelize their time for the advantage of all students.
- They should assess the potential of students only upon defined criterion and without any bias and discrimination.
- They should try to cultivate and grow the learning abilities of the students to a certain level before they leave the institute.
- They should try to ensure that all the students are gain sufficient quality knowledge as well as competence aligning with their core discipline after they finish their education.
- They should always encourage the students to develop their ultimate performance capabilities.
- They should facilitate and encourage group work and team work to consolidate newer approach.
- They should follow Blooms taxonomy in every part of the assessment.

Bloom's Taxonomy

• Students should take equal responsibility for implementing the OBE. Some of the responsibilities (not

Guidelines for Students

limited to) for the students in OBE system are as follows:

- • Students should be well aware of each UO before the start of a unit in each and every course.
- • Students should be well aware of each CO before the start of the course.
- • Students should be well aware of each PO before the start of the programme.
- • Students should think critically and reasonably with proper reflection and action.
- • Learning of the students should be connected and integrated with practical and real-life consequences.
- Students should be well aware of their competency at every level of OBE.

CONTENTS

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Physical World, Units and Measurements

UNIT SPECIFICS

This unit focusses on the following aspects of basic physics:

- Fundamental and derived physical quantities and units
- System of units
- Dimensional analysis of physical quantities its limitations
- Direct and indirect methods of measurement and least count of measuring instruments
- Estimation of errors and their propagation
- Significant figures

RATIONALE

Measurement of quantities is required in engineering, industry and daily life. The physical quantities can be measured. Units of quantities should be defined for measurement. Base physical quantities are independent and the rest are dependent on base quantities. The dimensional formula of a physical quantity is obtained by writing it in the terms of the base quantities. The method of dimensions is used for checking the correctness of equations and establish the relationship between quantities. The accurate value of a physical quantity can be found by determining measurement error. All these things will be studied in this text.

PRE-REQUISITES

Physics-Basic science

Mathematics–Basics algebra

Other-Basic knowledge of computer

UNIT OUTCOMES

- U1-O1: Identify physical quantities with their appropriate unit and determine the dimensions of the given physical quantities.
- U1-O2: Apply the method of dimensions and their limitations
- U1-O3: Describe the given measurement device and its application
- U1-O4: State with justification the error in the given measurement of a physical quantity.

Mapping of unit outcomes with the course outcomes:

11 **PHYSICAL QUANTITIES**

Interesting Facts

There are some interesting facts of units and dimensions such as a blue whale can weighs as much as 30 elephants, a cat can jump up to 7 times of their length. When water freezes it expands by 9%, crocodile swallow rocks to help them dive deeper.

$1.1.1$ **Physical Quantities**

The study of physics is based on measurement. It is necessary to measure various quantity to establish relationship among them. Those quantities which can be measured are known as physical quantities. For example : Length, mass, time, volume, speed etc.

On the other hand, loudness, the pitch are non-physical quantities since it is not possible to measure them. To measure a physical quantity, we require a standard unit. Each measurement consists of two parts, the first is numeric part (n) and the second is unit (u) of the physical quantity. Suppose mass of a block is 5kg, then $n=5$ is the numeric part which is 5 times of unit mass and u is the unit of mass i.e.kg. 5kg =5000g, thus, $n_1u_1=n_2u_2$ ($n_1=5$, $u_1=$ kg, $n_2=5000$ and $u_2=g$)

$1.1.2$ **Fundamental and Derived units**

In scientific work and daily life number of physical quantities are measured and each measured quantity needs to define a unit. However, a large number of quantities are interrelated. As for example, volume is related to length and speed is the ratio of length to time. Thus, there is no need to choose all quantities as independent quantities. Fundamental quantities are independent and rest quantities may be expressed in terms of the fundamental quantities. There are seven fundamental quantities and derived quantities are derived from these fundamental quantities.

Those quantities which are independent are called fundamental physics quantities. Units of fundamental quantities are called fundamental units. Fundamental quantities are also called base quantities. All other quantities that may be derived from fundamental quantities are called derived quantities and units of derived quantities are known as derived units. Speed, pressure, work and volume are some examples of derived quantities.

SI units: SI is abbreviated from the French name Le Systéme International d' Unités. In 1971 CGPM held its meeting and decided on this system of units which is called the international system of units.

SI units and name of seven fundamental quantities are given below in table (1.1) Table 1.1: Seven fundamental or base quantities

Table 1.2: Derived units

Besides the seven fundamental units, two supplementary units are defined.

$1.1.3$ **System of Units**

There are three systems of units used for measurement

- (1) F.P.S : (foot, pound and second) Here unit of length is foot, unit of mass is pound and unit of time is second
- (2) C.G.S : centimetre, gram, second (length in centimetres, mass in gram and time in second)
- (3) M.K.S : meter, kilogram, second (In this system of units, length is measured in meter, mass in kilogram and time in second)

In the SI system of units, all the seven fundamental quantities are measured in its extended form of the MKS system of units. In the SI system of units length is measured in metre, time in second, mass in kilogram, the temperature in kelvin, electric current in ampere, luminous intensity in candela and amount of substance is measured in the mole.

Power of 12 9 6 3 $\overline{2}$ $\mathbf{1}$ -1 -2 -3 -6 -9 -12 ten **Prefix** tera kilo hecto deca deci centi milli micro nano pico giga mega $\overline{1}$ K symbol G M h da d \mathbf{C} m μ n. p

Table 1.4: Prefixes of SI units, power of 10 and symbol of prefixes

Dimensions and Dimensional Formulae of Physical Quantities $1.1.4$

There are seven fundamental quantities and two supplementary quantities and all the rest quantities are derived quantities.

Derived quantities may be derived by the multiplication and division of base quantities. We can express a physical quantity in terms of base quantities. The power (or exponent) of a base quantity that enters into the expression is called the dimension of quantity in that base.

Dimensional formulae of physical quantities : force

Force = mass × acceleration = mass × $\frac{\text{length}}{(\text{time})^2}$ = MLT⁻² (dimensional formula of force is MLT⁻²)

(The dimensions of force are 1 in mass, 1 in length and -2 in time)

Volume = length x breadth x height = [L] [L] [L] = $M^0L^3T^0$ (dimensional formula and dimensions of volume are 3 in lengths and the rest are zero)

Density = $\frac{\text{mass}}{\text{volume}} = \frac{M}{I^3} = ML^{-3}T^0$

Note : To find dimensional formulae of physical quantities the magnitude is not considered. It depends on the equality of the type of quantity that enters. Thus, change in velocity, initial velocity, average velocity and final velocity all have the same dimensional formulae i.e., LT^{-1} .

Applications (Real Life / Industrial)

In daily life, the shopkeeper weighs the goods. When one goes to buy clothes, there is a need to measure the length of clothes. When a civil engineer builds a house, he maps and designs the house by measuring the length and breadth of the plot. The distance between the two cities is in kilometres and the fare for travelling by bus or air requires information in Rupees. In engineering and Industries measurements of different physical quantities are required.

Case-Study (Environmental / sustainability / social / ethical issues)

On April 15, 1999, a Korean Air Cargo Flight No. 6316 was on its way from Shanghai to Seoul. The crew members of the flight received the distance of the plane from ground in meters from the air traffic control (ATC) tower whereas the altimeter in the airplane showed the distance in feet. Because the crew members misunderstood the units while calculating the distance, the plane strayed from its path and fell to the ground. As a result three crew members and 5 others were killed and 37 were injured in the accident. This case study shows that it is necessary to take care of the unit alongwith its value in measuring any physical quantity.

Create Inquisitiveness and Curiosity

y = asin $\frac{2\pi}{T}$ is an equation of periodic motion. Find printing error in this equation. Here 'a' is maximum displacement.

Solved Problems

Problem-1: Find dimensional formulae of the following quantities: work, power and torque. Solution:

(a) Work = force \times displacement $W = F \times length = MLT^{-2}$. $L = ML^{2}T^{-2}$

(b) Power =
$$
\frac{\text{work}}{\text{time}}
$$

P = $\frac{\text{work}}{\text{t}} = \frac{\text{ML}^2 \text{T}^{-2}}{\text{T}} = \text{ML}^2 \text{T}^{-3}$

 (c) Torque = force \times prependicular distance from the force $\tau = F \times length = MLT^{-2}.L = ML^{2}T^{-2}$

Problem-2: Find dimensional formulae of the following quantities given below.

- (a) the young's modulus of elasticity (Y)
- the coefficient of viscosity (η) (b)
- the universal constant of gravitation (G) (c)

$$
Y = \frac{mgL}{\pi r^2 l}
$$
, $F = \frac{Gm_1m_2}{r^2}$ where the symbols have their usual meaning

Solution:

(a)
$$
Y = \frac{mgL}{\pi r^2 l}
$$

\n $[Y] = \frac{MLT^{-2}L}{L^2 L} = ML^{-1}T^{-2}$

(b) Viscous force
$$
F = 6\pi r v \eta
$$

$$
[\eta]=\frac{MLT^{-2}}{L.LT^{-1}}=ML^{-1}T^{-1}
$$

(c)
$$
F = \frac{Gm_1m_2}{r^2}
$$

\n $G = \frac{Fr^2}{m_1m_2} = \frac{MLT^{-2}L^2}{M^2} = M^{-1}L^3T^{-2}$

$1.2₂$ **DIMENSIONAL ANALYSIS**

Interesting Facts

A light-year is a unit of distance travelled by light in vacuum in one year. Minus 40° C temperature is equal to minus 40° F.

$1.2.1$ **Principle of Homogeneity of Dimensions**

This principle states that physical quantities of the same dimensions can add or subtract. For example, mass cannot be subtracted from length and work cannot be added to velocity. In other words, all the terms in an equation must be dimensionally equal (or homogeneous). so, one can use this principle of homogeneity to check the correctness of the given equation.

Dimensional Equation and Their Applications $1.2.2$

A. Checking correctness of dimensional equations

Using the method of dimensions one can check the correctness of an equation whether it is dimensionally correct or not.

Taking an example

Let us check the equation

 $v^2 = u^2 + 2ax$ whether it is dimensionally correct or not.

Solution:

Here, v is the final velocity of a body which starts with an initial velocity u and has an acceleration a along the direction of motion, x is the displacement travelled by the body.

$$
[v^2] = (\text{final velocity})^2 = \left(\frac{\text{length}}{\text{time}}\right)^2 = L^2 T^{-2}
$$

$$
[u^2] = (\text{initial velocity})^2 = \left(\frac{\text{length}}{\text{time}}\right)^2 = L^2 T^{-2}
$$

$$
a.x = \text{acceleration} \times \text{displacement}
$$

$$
= \frac{\text{velocity}}{\text{time}} \times \text{displacement}
$$

$$
= \frac{\text{length } / \text{ time}}{\text{time}} \times \text{length} = L^2 T^{-2}
$$

Here, the dimensions of all three terms are the same thus the equation may be correct based on dimensions.

Example: Check the correctness of formula F = $\frac{mv^2}{r}$

Where F is force, m is mass, v is the velocity of particle and r is the radius of a circle.

Solution:

The dimension of the velocity is LT^{-1}

Thus, the dimension of the right-hand side is $\frac{M.(LT^{-1})^2}{I} = MLT^{-2}$

The left-hand side is a force so the dimension is MLT⁻²

The dimensions of both sides are equal, thus, the formula may be correct.

Conversion from one system of the unit to other В.

By using dimensions one can convert numeric part of the unit of a physical quantity from one system to other.

Consider an example : Convert one-joule work into ergs(CGS).

Let us first write dimensional formula of work

 $W = F.x = Force \times displacement$

$$
[W] = [F][x] = MLT^{-2} \cdot L = ML^{2}T^{-2} \text{ here } n_{1} = 1, u_{1} = \text{joule (MKS)}, u_{2} = CGS \text{ unit}, n_{2} = ?
$$

$$
n_{1}u_{1} = n_{2}u_{2}, \text{ thus, } n_{2} = \frac{n_{1}u_{1}}{u_{2}} = 1 \left(\frac{1 \text{ kg}}{1 \text{ g}}\right) \left(\frac{1 \text{ m}}{1 \text{ cm}}\right)^{2} \left(\frac{1 \text{ s}}{1 \text{ s}}\right)^{-2} = \left(\frac{1000 \text{ g}}{1 \text{ g}}\right) \left(\frac{100 \text{ cm}}{1 \text{ cm}}\right)^{2} \left(\frac{1 \text{ s}}{1 \text{ s}}\right)^{-2}
$$

 $= 10^3 \times 10^4 = 10^7$ ergs

1 joule = 10^7 ergs

One can convert a physical quantity from one system to another if the dimensional formula of that derived quantity is known.

Deducing relation between the physical quantities \mathcal{C} .

Sometimes relation between given physical quantities can be deduced using the method of dimensions. In this method to establish the relation between given quantities, one can assume that dependence among physical quantities is of product type. This application of dimensions can be understood by taking an example.

Example:

Suppose we want to derive the expression for the frequency of a stretched string that vibrates due to tension (F) in string and the frequency may depend on the applied tension (F), length (I) of the string, and mass per unit length (μ) of the string.

Solution:

We assume that the dependence of frequency on these quantities is of product type, that is

 $n = k l^a F^b \mu^c$ $...(1.1)$

where, k is dimensionless constant and a, b, and c are exponents which we want to determine. By considering dimensions on both sides, we have

 $T^{-1} = L^a (MLT^{-2})^b (ML^{-1})^c = L^{a+b-c} M^{b+c} T^{-2b}$

Since the dimensions on both sides must be equal, we have

 $a + b - c = 0$; $b + c = 0$: $-2b = -1$

And after solving b = $\frac{1}{2}$, c = $-\frac{1}{2}$, a = -1

Putting these values in equation (1.1)

$$
n = \frac{K}{l} \sqrt{\frac{F}{\mu}}
$$
...(1.2)

Thus, by dimensional analysis, we can establish a relationship that the frequency of the stretched string is proportional to the square root of the tension in the string, and is inversely proportional to the length of string and the square root of mass per unit length of the string. Constant $k = \frac{1}{2}$ determined by some other method but cannot be found by the dimensional analysis.

$1.2.3$ **Limitations of Dimensional Analysis**

Although applications of the dimensional equation are useful there are some limitations of dimensional analysis.

- 1. A relation can be deduced between given physical quantities only if the dependence is on the product type. For example: $v^2 = u^2 - 2ax$, cannot be deduced.
- $2.$ This method cannot be used if a particular physical quantity depends on more than three quantities because there will be more unknowns but fewer equations. Thus, exponents can't be found uniquely.

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- The dimensionless numerical constant cannot be determined by the method of dimensional $\overline{3}$. analysis.
- Equations can't be deduced which contain trigonometrical terms ($sin(x)$, $cos(x)$, $tan(x)$ etc.), 4. exponential functions (e^x , a^x etc.) and logarithmic terms ($log(x)$).

Applications (Real Life/Industrial)

Physical quantities need to be converted from one unit to another such as price of 1 kg of sugar is ₹40, then the price of 250 grams of sugar has to be determined. If the length and breadth of the plot are given in feet, it needs to be converted into meters. The civil engineer provides the weight of iron in tones and it needs to be converted into kilograms.

Case-Study (Environmental /sustainability/social/ethical issues)

NASA Mars Climate Orbiter was accidentally destroyed on a mission of mars, instead of entering orbit in September 1999. The cause of the failure of the mission was a miscommunication of unit of force which was used differently (newton versus pound-force) in different computer programs. As a result considerable amount of effort, money and time was wasted.

Create Inquisitiveness and Curiosity

Taking force, mass and time to be fundamental quantities determine the dimensions of (a) torque (b) pressure (c) acceleration.

Solved Problems

Problem-1: Check correctness of the equation $x = ut + \frac{1}{2}at^2$ based on dimensional analysis.

Solution: $[x] = \text{length} = L$, $[ut] = \text{velocity} \times \text{time} = LT^{-1} \cdot T = L$ and $[at^2] = \text{acceleration} \times \text{time}^2$ $= LT^{-2}.T^{2} = L$

The dimensions of all these terms are equal thus, the given equation may be correct.

Problem-2: In a simple pendulum a bob of mass (m) is attached to a string of length (l) and it oscillates under gravity with time period (T). If the time period depends on the length of string (l) , the mass of bob m and acceleration due to gravity g, deduce the relation of time period with given quantities using the method of dimensional analysis.

Solution: Suppose dependence of time period on quantities *l*, m and g may be product type which can be written as T = $k l^a$ m^b g^c $...(1)$

Where, k is dimensionless constant and a, b, c are exponents.

Taking dimensions we have, $T = [L]^a [M]^b [LT^{-2}]^c$: $L^0 M^0 T^1 = L^{a+c} M^b T^{-2c}$

in power of M, L and T are equal on both sides, we have $a + c = 0$, $b = 0$ and $-2c = 1$

so, $a = \frac{1}{2}$, $b = 0$, $c = -\frac{1}{2}$. Substituting these values of a, b and c in equation (1),

T =
$$
kI^{1/2}
$$
 g^{-1/2} i.e., T = $k\sqrt{\frac{I}{g}}$

The value of constant k determined by some other method is 2π , here the value of k can't be found by dimensional analysis.

1.3 MEASUREMENTS

Interesting Facts

Do you know your palm size can also work as a unit although not as accurately as standard measuring instruments? Check the dimensions of the refrigerator in your home in palm unit and compare with standard units

$1.3.1$ **Measurements**

Measurement is a process or set of operations to compare two physical quantities, one which has unknown magnitude with the other which is a predefined standard quantity (or unit). Measurement has evolved due to invented instruments or by instruments themselves and its progress is fuelled by the need to master the surrounding environment; to master the means of production for fetching daily food to making some arrangements to sustain the cold weather; to create money and to move trade dynamics; to master and control the energy of various forms and at present to master the information using latest devices.

$1.3.2$ **What is a Measuring Instrument?**

It is an instrument that shows the degree or the extent or the quantity of something that we normally observe around us. There are numerous measuring instruments as Vernier Calipers, Micrometer Screw Gauge, Dial Gauge, Radius Gauge, Surveying compass, Multimeter, Electronic sensors, Flowmeter etc. for various engineering applications.

1.3.3 Least Count of a Measuring Instrument and Zero Errors

The smallest measurement that can be accurately taken by any measuring instrument is called its least count (L.C.). It gives the resolution of a measuring instrument.

For example,

- (a) The simple ruler can measure 0.1 cm or 1mm accurately, which is its least count, or in other words, its resolution.
- (b) Least count of Vernier Calipers is typically $0.01 \text{cm} = 0.1 \text{ mm}$. So, comparing the least count of a ruler and a Vernier Calipers, one can say that the latter has a better resolution.

The least Count (L.C) of Vernier Calipers is given by the following formula:

(c) Micrometer Screw Gauge normally in labs measures up to 0.001 cm or 0.01 mm or even less. The least Count (L.C) of Micrometer Screw Gauge is given by the following formula:

Least count of micrometer screw gauge

Pitch of a screw gauge

Number of divisions on circular scale

Here, the pitch of a screw gauge is defined as the distance between two consecutive threads of a screw.

(d) For measuring the radius of curvature of convex or concave surfaces or lenses, one can use Spherometer, which has the least count typically up to 0.001 cm or 0.01 mm.

Least Count of spherometer = $\frac{\text{Pitch of the spherometer screw}}{\text{Number of divisions on circular scale}}$

Each of these instruments needs to be checked for zero error before using them for any measurements. More details about the determination of zero error of these instruments are available in respective experiments.

$1.3.4$ **Types of Measurement**

There are two types of dimensional measurements: (a) Direct and (2) Indirect or Inferential. Measuring instruments such as Vernier Calipers, Micrometer Screw Gauges, and coordinate measuring machines are direct measurement instruments and are used to measure the dimensions of the target directly. Direct measurements are simple, almost straightforward and economical. Indirect measurements are also known as comparative measurements because comparison of targets with standard dimensions of reference devices is required. This method produces the output for remote processing and employs the very latest technology in its measurement.

Applications (Real Life / Industrial)

In chemical industries, the plant operator takes correct and necessary steps for the smooth functioning of chemical plants by measuring temperatures and pressures at various points that shows the progress of chemical reactions.

Case-Study (Environmental/Sustainability/Social/Ethical Issues)

If a tyre of your vehicle is under-inflated by 5 psi (pounds per square inch) it can reduce its life by around 25%. Under-inflated tyres can increase fuel consumption by about 5% that harms the nature, environment and your wallet (!).

Create Inquisitiveness and Curiosity

What are the least count and the range of your wristwatch? what about your friend's wristwatch? Compare your results among your group of friends.

Solved Problems

Problem-1: If L.C of micrometre screw gauge is 0.01mm and there are 50 divisions on the circular scale and the zero error is $+0.02$ mm, then find the total corrected value for the thickness of an A4 size paper. The main scale reading is 0.70 mm and the circular scale reading is 7 divisions.

Solution:

Here, L.C. of micrometer screw gauge = 0.01 mm and zero error (e) = + 0.02 mm Therefore, the correction $(c) = -0.02$ mm Main scale reading (MSR) = 0.70 mm; Circular scale reading (CSR) = 7 div. .. Total reading (TR) in mm = (MSR + CSR x LC)= [0.70 + (7 × 0.01)] mm = 0.77 mm Corrected value of observation = Total reading \pm Correction Hence, corrected value = $(0.77 - 0.02) = 0.75$ mm

ERRORS IN MEASUREMENTS 1.4

Interesting Facts

If you are playing soccer and you always hit the right goalpost instead of scoring, then you are not accurate, but you are precise! If you are playing darts and you always hit at different spots inside the outermost ring while missing out on the target (i.e., centre), you are accurate but not precise!

$1.4.1$ **Errors and Types of Errors**

Errors are the uncertainty in the result of every measurement by any measuring instrument. Measurement errors are often called observational errors.

The accuracy of measurement shows how close the measured value is to the true value of the quantity.

Precision gives us resolution or limits up to which the quantity is measured. Every measurement gives approximate values due to measurement errors. In general, measurement errors can be broadly classified as (1) systematic errors and (2) random errors.

- (1) Systematic Errors: Systematic errors are unidirectional, either positive or negative. They are classified in further 3 categories: (a) Instrumental errors (b) Imperfection in experimental technique or procedure (c) Personal errors.
	- (a) Instrumental errors: They arise from the errors due to faulty and imperfect design or calibration of the measuring instrument, zero error in the instrument, etc.
	- (b) Imperfection in experimental technique or procedure: Unstructured procedure or technique or other external conditions (such as changes in temperature, humidity, pressure, altitude, wind velocity, etc.) during the experiment may affect the measurement.
	- (c) Personal error: arise due to lack of proper experimental and apparatus set-up, individual's inexperience or carelessness in taking observations without observing proper precautions, etc. Systematic errors can be minimised systematically by improving techniques or procedures, selecting better and more precise instruments, and removing personal errors as far as possible.
- (2) Random errors: Random errors are those errors, which occur at irregular periods and therefore, are random for sign and size. These errors can arise due to random and unpredictable fluctuations in experimental conditions (e.g., unpredictable fluctuations in temperature, pressure, voltage supply etc.), personal errors by the observer in taking readings, etc.

$1.4.2$ **Estimation of Errors in Measurements**

(a) Absolute Error $(|\Delta a|)$: Suppose in repeated measurements of the physical quantity, the values obtained are a_1 , a_2 , a_3, a_n . Then the arithmetic mean of these values is given as : $\overline{a} = a_{\text{mean}} = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}$. In absence of a true value, arithmetic mean is considered as the true value. Then the errors in the individual measurement are written as, $\Delta a_1 = \overline{a} - a_1$; $\Delta a_2 = \overline{a} - a_2$; $\Delta a_3 = \overline{a} - a_3$, $\Delta a_n = \overline{a} - a_n$

Errors in individual observation may be positive or negative but their absolute error $|\Delta a|$ will always be positive.

- **(b)** Mean Absolute Error $(\overline{|\Delta a|})$: It is represented by $\Delta \overline{a}$ or Δa_{mean} . Thus, $|\overline{\Delta a}| = \overline{\Delta a}_{\text{mean}} =$ $\frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + + |\Delta a_n|}{n}$. For a single measurement of the value, 'a' can be written as $a = a_{mean} \pm \Delta a_{mean}$.
- (c) Relative Error (or Fractional Error) (δ a) and Percentage Error (δ a%) :

Relative error (δa) = $\frac{\overline{\Delta a}}{\overline{a}} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$ and percentage error, $\delta a\% = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$

Error Propagation in Arithmetic Operations $1.4.3$

Suppose two physical quantities X and Y with their measured values and errors are written as $X \pm \Delta X$, Y $\pm \Delta Y$, respectively. Here, ΔX and ΔY are their absolute errors. Let Z be the resultant quantity and ΔZ be its absolute error.

- (a) Errors in a summation $(Z = X + Y)$ or a subtraction $(Z = X Y)$: The maximum possible error in Z for either summation or subtraction is given by, $\Delta Z = \Delta X + \Delta Y$. The absolute error in the final results of summation or difference of the quantities is always added.
- (b) Errors in a multiplication $(Z = XY)$ and a division $(Z = X/Y)$: The maximum relative or fractional error in either multiplication or division is given by $\frac{\Delta Z}{Z} = \frac{\Delta X}{X} + \frac{\Delta Y}{Y}$. The relative or fractional error in the final results of multiplication or division of quantities are always added.
- (c) Error in a quantity with powers: Suppose, $Z = k \frac{X^n Y^m}{C^q}$ where, k = constant. Then the relative or fractional error can be written as $\frac{\Delta Z}{Z} = n \frac{\Delta X}{Y} + m \frac{\Delta Y}{Y} + q \frac{\Delta C}{C}$. The relative error or fractional error in a physical quantity raised to some power is the power times the relative or fractional error in the individual quantity. The constant multiplier is not considered for the evaluation of relative error.

Significant Figures or Numbers $1.4.4$

In a reported measurement, the reliable digits plus the first uncertain digit are known as significant digits or significant figures. Significant figures show the precision of measurement that depends on the least count of the measuring instrument.

Rules for determining the significant figures

- (1) All the non-zero digits (i.e., digits having numbers $[1-9]$) are significant.
- (2) All the zeros between two non-zero digits are significant irrespective of where the decimal point is placed.
- (3) If the number is less than $\langle \langle \rangle$ 1 (one), the zero(s) on the right of the decimal point are significant but the zero(s) to the left of the first non-zero digit are not significant. Also, note that the
trailing zero(s) in a number with a decimal point are significant. [For e.g., In 0.001430 , the underlined zeroes are not significant.

- (4) If the number has the terminal or trailing zero(s) without a decimal point, then these zero(s) are not significant. [For e.g., 139 m = 13900 cm = 139000 mm has three significant figures, the trailing $zero(s)$ being not significant.].
- (5) To remove confusions in determining the number of significant figures for values with trailing $zero(s)$, the safest way to report every measurement in scientific notation (in the power of 10). [For e.g., $100.0 = 1.0 \times 10 E+2$]

Rules for Arithmetic Operations with Significant Figures

In multiplication or division, the least significant figures present among all the original numbers should be considered in the final result. In summation or difference, the final result retains the least decimal places present among all the original numbers.

Applications (Real Life / Industrial)

Almost every human activity involves the use of measurements and errors to some extent. E.g., the applications of measurements and error analysis are realized in GPS or GNSS Technology, in the medical and health sector, in various Olympic level sports activities, manufacturing and production industries.

Case-Study (Environmental/Sustainability/Social/Ethical Issues)

Air Canada flight 143 used up all its fuel halfway to its destination from Montreal to Edmonton at an altitude of 41,000 feet on July 23, 1983, leading to its emergency landing safely. The incident occurred due to a conversion error in the measurement of the fuel requirement of the flight. In place of using kg/ lit., an imperial unit system was used.

Create Inquisitiveness and Curiosity

The degree of accuracy is half a unit on each side of the measurement units. A wood that measures 5.5 meters long, accurate to or nearest to 0.1 of a meter means it could be anywhere between 5.45 m and 5.55 m long. What is the accuracy in your height in inches?

Solved Problems

Problem-1: If the diameter of a ball of the ballpoint pen is given by (0.70 ± 0.01) mm. What does it mean?

Solution: It means that the true value of the diameter of a ball of the ballpoint pen is lying between 0.69 mm and 0.71 mm .

Problem-2: If all measurements in an experiment are performed up to the same number of times, then due to which measurement does a maximum error occur?

Solution: Maximum error occurs due to the measurement of the quantity that appears with maximum power in the formula. If all the quantities in the formula have the same powers, then a maximum error occurs due to the measurement of the quantity whose magnitude is the least.

Problem-3: If the radius of a sphere is measured with the micrometer screw gauge and the observations are as following, find out the percentage error in measurement of the radius of the sphere: $R_1 = 2.46$ cm, $R_2 = 2.40$ cm, $R_3 = 2.48$ cm, $R_4 = 2.43$ cm, $R_5 = 2.42$ cm.

Solution:

Mean of observations $=$ (i)

$$
\overline{a} = \frac{a_1 + a_2 + a_3 + a_4 + a_5}{5} = (2.46 + 2.40 + 2.48 + 2.43 + 2.42)/5 = (12.19)/5 = 2.438
$$
 cm

- (ii) Absolute Errors in individual observation: $\Delta a_1 = \overline{a} - a_1$; $\Delta a_2 = \overline{a} - a_2$; $\Delta a_3 = \overline{a} - a_3$; $\Delta a_4 = \overline{a} - a_4$; $\Delta a_5 = \overline{a} - a_5$ $|\Delta a_1|$ = | 2.438 - 2.46 | = | -0.022 | cm = 0.022 cm; $|\Delta a_2|$ = | 2.438 - 2.40 | = | +0.038 | cm = 0.038 cm; $|\Delta a_3|$ = | 2.438 - 2.48 | = | -0.042 | cm = 0.042 cm; $|\Delta a_4|$ = | 2.438 - 2.43 | = | +0.008 | cm = 0.008 cm; $|\Delta a_5|$ = | 2.438 - 2.42 | = | +0.018 | cm = 0.018 cm;
- (iii) Mean of Absolute Error = $\Delta \bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + |\Delta a_4| + |\Delta a_5|}{5} = (0.022 + 0.038 + 0.038 + 0.038)$ $0.042 + 0.008 + 0.018$ / $5 = 0.128$ / $5 = 0.0256$ cm

(iii) Relative Error =
$$
\delta \mathbf{a} = \frac{\Delta \overline{a}}{a} = \frac{\text{Mean Absolute Error}}{\text{Mean of the observations}} = (0.0256 \text{ cm}/2.438 \text{cm}) = 0.01
$$

(iv) Percentage Error =
$$
\delta a \mathcal{L} = \frac{\Delta \bar{a}}{a} \times 100\% = (0.01 \times 100) \% = 1.0\%
$$

Problem-4: Find the number of significant figures in the following observations:

(a) 0.001sec (B) 4.34 x 10⁴ m/s (c) 0.5250 kg (d) 6.0780 N/m² (e) 31.052 calorie (f) 0.0009012 m (g) 1.0203 x 10^{12} Hz (h) 1.5 x10⁻⁹ ampere

Solution: Number of significant figures in following observations are:

UNIT SUMMARY

- Measurable and observable quantities are called physical quantities.
- Seven fundamental quantities are independent. There are two supplementary quantities plane angle and solid angle. Unit of which are radian (rad) and steradian (sr), respectively.
- Derived quantities are derived from base quantities and units of derived quantities are called derived units.
- Dimensions of physical quantities are powers to which base quantities are raised to obtain the unit of that physical quantity.
- The dimensional formula of a physical quantity is an expression in terms of seven base quantities.
- Methods of dimension can be used to convert the unit from one system to other, checking of dimensional equation and derivation of the equation.
- Least count is the smallest value that can be measured by any measuring instrument. Measured values are accurate only up to this value.
- The choice of measuring equipment depends on the degree of accuracy required.
- Accuracy means correct value (i.e., hitting a bullseye). Precision means repeating (i.e., hitting the same spot, but maybe not the correct spot.)
- Significant figures show the precision of measurement that depends on the least count of the measuring instrument.

EXERCISES

(A) **Objective Questions**

The equation of a real gas is given by $(P + \frac{a}{v^2}) (V - b) = RT$. Where, P, V and T are pressure, volume and temperature respectively and R is the universal gas constant. The dimensions of

1.10 The length of a smartphone is 6 inches. Measuring the length of a table in a smartphone length unit, _____ smartphone length units will be equal to 91.44 cm length of a table. (9, 6, 11, 15)

[LOD2]

 $[LOD2]$

(A) Answers of Objective Questions

resistance is $(.43.6\%, ±3.8\%, ±3.3\%, ±3.5\%)$

A1.1. (b) The dimensions of impulse are: Impulse = force \times time = MLT⁻². T = MLT⁻¹

A1.2. (d) Stress =
$$
\frac{\text{Force}}{\text{area}} = \frac{\text{MLT}^{-2}}{L^2} = \text{ML}^{-1}\text{T}^{-2}
$$
, Toung's modulus $Y = \frac{\text{mgl}}{\pi r^2 l} = \frac{\text{MLT}^{-2}L}{L^3} = \text{ML}^{-1}\text{T}^{-2}$
Pressure = $\frac{\text{Force}}{\text{area}} = \text{ML}^{-1}\text{T}^{-2}$ Power = $\frac{\text{work}}{\text{time}} = \text{ML}^{2}\text{T}^{-3}$

A1.3. (d) Angular momentum = linear momentum \times length = kg m/s \times m = kgm²s⁻¹

A1.4. (b) Energy density = $\frac{\text{Energy}}{\text{volume}}$ = ML⁻¹T⁻², Force = mass × acceleration = MLT⁻²

$$
Pressure = \frac{Force}{area} = ML^{-1}T^{-2}, Velocity = LT^{-1}
$$

Linear momentum = mass \times velocity = MLT⁻¹

A1.5. (a) Dimensional quantity may have unit.

For example, plane angle = $\frac{\text{arc}}{\text{radius}}$ = M⁰L⁰T⁰ dimensionless but unit of angle is radian.

A1.6. $p + a/v^2$ means addition of two terms so, dimension of these two terms should have same $p =$

$$
a/v^2
$$
, $a = pv^2 = \frac{Force}{area} \times volume^2 = ML^5T^{-2}$

- A1.8. 1.042 x 10²³ [\cdot (1.0305 +0.011) × 10²³ = 1.0415 × 10²³ but for addition we take the least decimal places in original numbers, therefore, answer is 1.042×10^{23} after rounding-off.]
- A1.9. \pm 3.8% [\cdot : R_{eq} $\pm \Delta R_{eq} = (R_1 \pm \Delta R_1) + (R_2 \pm \Delta R_2) + (R_3 \pm \Delta R_3) = (10 \pm 1\%) + (12 \pm 0.8\%) +$ $(20 \pm 2\%) = (10 + 12 + 20) \pm (1 + 0.8 + 2)$ % = 42 ± 3.8% ohms
- A1.10.6 [\cdot Length of a table is 91.44 cm; Since 2.54 cm = 1 inch, 91.44 cm = 36 inches. Here, 6 inches $= 1$ smartphone lengths, \therefore 36 inches $= 6$ smartphone lengths

Subjective Questions (B)

 1.1 Find the dimensions of

(a) force

(b) electric potential

- (c) linear momentum
- 1.2 State the principle of homogeneity of dimensions and using dimensional analysis deduce relation for the force exerted on a body of mass 'm' and acceleration of body due to force is 'a'. [LOD3]
- 1.3 Find the dimensions of plank's constant h, when energy given by an equation, $E = hf$ and f is the frequency. [LOD1]
- 1.4 Check, the correctness of the following equations based on dimensional analysis.
	- (b) $v = \sqrt{\frac{T}{m}}$ (a) $S = \frac{\rho r h g}{2 \cos \theta}$
	- (c) $F = 6\pi r v \eta$

Where h is height, S is surface tension, ρ is density, T is tension, m is mass per unit length and n is coefficient of viscosity. $[LOD2]$

1.5 Convert power of a motor of 1500watt in CGS unit.

1.6 Determine the dimension of A and k from the equation of progressive wave,
$$
y = Asin (\omega t - kx)
$$

where, $y = displacement$, $\omega = \frac{2\pi}{T}$ angular velocity, $t = time$, of the wave. [LOD3]

- 1.7 Give reasons to use a micrometer screw gauge over a vernier Calipers for precise measurements. [LOD1]
- 1.8 If the main scale reading is 1.3 cm and the vernier scale reading is 21 divisions, find the total reading and corrected reading for Vernier Calipers whose least count is 0.02 mm. Negative Zero error in Vernier Calipers is 2 divisions. $[LOD1]$
- 1.9 A voltmeter with a range of $0 - 250$ volts is connected in a circuit to measure the voltage drop across a component, and after switching it ON and OFF a few times, it consistently displays 5 volts. How will you deal with this situation to measure the data as accurately as possible?

[LOD2]

 $[LODI]$

 $[LOD2]$

1.10 I have a piece of a thread of length 10 cm, and a copper rod of the same diameter and a ruler scale. What I don't have is measuring instruments like Vernier Calipers, or micrometer screw gauge. How can I measure the diameter of a rod to an accuracy of 1mm? The range of a ruler scale is 0-30 cm. $[LOD3]$

Answers of Subjective Questions (B)

A1.1. (a) MLT⁻² (b) ML²T⁻³I⁻¹ (c) MLT⁻¹

A1.2. $F = Kma$ $K=1$ (value of constant determine by the experiment)

A1.3. $[h] = ML^{2}T^{-1}$

- A1.4. (a) may be correct (b) may be correct (c) may be correct
- A1.5. 1.5×10^{10} erg/s (CGS unit)
- A1.6. $[A] = L, [K] = L^{-1}$
- A1.8. 13.46 mm = 1.346 cm [... Total reading = MSR + (VSR x LC) = [13 + (21 x 0.02)] mm = 1.342 mm or 1.342 cm; Corrected reading = Total reading + correction = $13.42 +$ $(2 \times 0.02) = 13.42 + 0.04 = 13.46$ mm

PRACTICALS

1. To measure the length, the radius of a given cylinder, a test tube and a beaker using a Vernier caliper and find the volume of each object.

Practical significance

Students will be using vernier caliper in the measurement of outer diameter, depth, inner diameter and length of objects during studies and also in their working profession. In industries, normal scales are useless when there is a need to measure dimensions of objects less than 1mm In such kind of measurement vernier caliper is used to measure dimensions up to 1mm with accuracy. In this experiment use of a vernier caliper is explained very well.

Relevant theory

Vernier calliper is a simple instrument by which the length of an object can be measured accurately up to one-tenth of a millimetre or one-hundredth of a centimetre.

The formula used: Least count of vernier caliper

the magnitude of the smallest division on the main scale lmm $= 0.1$ mm = 0.01cm

- $L.C. =$ number of division on the vernier scale
- (a) Observed Reading = Main scale reading + (vernier division coincide with main scale \times L.C)
- (b) Corrected Reading = Observed reading $-$ (\pm Zero error)
- (c) The volume of a cylindrical object $V = \pi r^2 h$

where h is its length, r is the internal radius of a cylindrical object.

Practical outcomes (PrO)

PrO1: Determine the least count (L.C.) and zero error of Vernier Caliper

PrO2: Use Vernier Caliper in the measurement of length, internal diameter and depth of objects PrO3: Determine the volume of given objects

Practical setup (drawing/sketch/circuit diagram/work situation)

Fig.1.1 : Vernier Caliper :

 $l =$ jaws for measuring internal diameter, $Q =$ jaws for measuring outer diameter, $V =$ vernier scale, M = main scale, D = stem or depth rod, R = thumb screw, L = Lock screw

Resources required

Precautions

- 1. If the vernier scale is not sliding smoothly over the main scale, apply machine oil/grease.
- 2.5 Tighten the screw of the vernier without exerting undue pressure to avoid any damage to the threads of the screw.
- $3.$ Keep the eye directly over the division mark to avoid any error due to parallax.
- $4.$ Note down each observation should have corrected significant figures and units

Suggested procedure

(A) Measuring the diameter of a cylindrical body

- $\mathbf{1}$. Identify each part of vernier caliper
- $\overline{2}$. Determine the least count of vernier caliper by given formula as above in relevant theory
- $3.$ Calculate zero error as a given method with (Fig. 1.2 to Fig.1.4)
- $4.$ Now hold the cylindrical object between jaws for measuring outer diameter O. Note the reading of the main scale immediately to the left of the zero point of vernier scale.
- 5. Observe the coincidence of a vernier scale division with that of a main scale division in the vernier window from the left end (zero) to the right (Position your eye directly over the division mark to avoid any parallax error). It is a vernier scale reading denoted by n.
- 6. Total reading = Main scale reading + $n \times L.C.$
- 7. Repeat steps 4 to 6 to obtain the diameter of the object at a position perpendicular to the above positions. Take three sets of reading in each case.
- 8. Find the mean of the corrected readings of the diameter of the given cylindrical object and finally calculate the volume above by putting the values in the given formula
- (B) Measuring the internal diameter and depth of the given beaker and test tube to find the volume.
	- 1. Adjust the upper jaws I of the Vernier Caliper to touch the wall of the beaker from inside without exerting undue pressure on it. Tighten the screw gently to fix the position.
	- $2.$ Repeat steps 4 to 6 as in (A) to obtain the value of the internal diameter of the beaker. Do this for three different positions of the beaker
- To determine the depth of the beaker, keep the edge of the main scale of Vernier Caliper on its $3.$ peripheral edge.
- Keep sliding the moving jaw of the Vernier Caliper until the metallic strip or stem D just touches 4. the bottom of the beaker and it should be perfectly perpendicular to the bottom surface. Now tighten the screw of the Vernier Caliper.
- $5.$ Now again find the reading of the main scale and coincide division of vernier scale
- Then total reading=main scale reading $+ n \times L.C$ of the experiment to obtain the depth of the 6. given beaker. Take the readings for depth at different positions of the beaker.
- $7.$ Record the observations and apply zero corrections, if required. Calculate the mean of the corrected readings of the internal diameter and depth of the beaker and finally calculate the volume above by putting the values in the given formula.
- Repeat the above same process from $[(B) 1 to 7]$ for the test tube also. 8.

Observations and calculations

Observations

(1) Least count of vernier caliper (vernier constant) Smallest division on main scale (MSD) = \times cm Total number of divisions on vernier scale = N

Least count =
$$
\frac{\text{MSD}}{\text{N}} = \frac{\text{X}}{\text{N}} =
$$
 ______ cm

- (2) Length of cylindrical object =cm
- (3) Zero error and its correction

When the jaws measuring outer diameter O touch each other, the zero of the vernier should coincide with the zero of the main scale then, there is no error in the instrument as shown in $(Fig.1.2)$ If it is not so, the instrument is said to possess zero error (This might have happened due to a manufacturing defect or due to rough handling). Zero error may be positive or negative, depending upon whether the zero of vernier scale lies to the right or the left of the zero of the main scale as shown in (Fig.1.3 & Fig.1.4). In this situation, a correction must be required to the observed reading.

(a) Positive zero error

When both jaws are touching each other, zero of the vernier scale is shifted to the right of zero of the main scale see in (Fig.1.3). In this situation, during measurements, the reading taken will be more than the actual reading. Hence, zero error is positive in this case and value of positive zero error $= 0.0 + 6 \times 0.01 = (+) 0.06$ cm. For any measurement done, the zero error should be 'subtracted' from the observed reading. Thus, corrected reading = observed reading $- (+$ zero error)

(b) Negative zero error

When both the jaws are touching each other, zero of the vernier scale is shifted to the left of zero of the main scale see in (Fig.1.4). This situation makes it obvious that while taking measurements, the reading taken will be less than the actual reading. Hence, zero error is negative in this case and value of negative zero error = $0.0 + 5 \times 0.01 = (-) 0.05$ cm. Thus, corrected reading = observed reading $-$ (– zero error)

Observation table

Measuring the diameter of a cylindrical object a.

Measuring the internal diameter and depth of a test tube \mathbf{b} .

NOTE: Draw the same table again for the diameter and depth of a beaker

Results and/or interpretation

- Length of cylindrical object $(l) =$ cm a.
- \mathbf{b} . Mean corrected diameter of cylindrical object $(D) = ...$ cm
- Mean corrected radius of cylindrical object (R) = $\frac{D}{2}$ =cm c.
- Volume of cylindrical object $(V_1) = \pi R^2 l =$cm³ d .
- Mean corrected internal diameter of the test tube $(d) =$cm e.
- f.
- Depth of test tube $(l_1) =$cm g.
- Volume of test tube $(V_2) = \pi r^2 l_1 = \dots \dots \dots \dots$ h.
- Mean corrected internal diameter of beaker(d') =.............cm i.
- Mean corrected radius of beaker(r') = $\frac{d'}{2}$ =.........cm j.
- k. Depth of beaker $(l_2) =$cm
- \mathbf{l} . The volume of the beaker $(V_2) = \pi r^2 l_2$

Conclusions and/or validation

(To be filled by student)

Practical related questions or viva-voce questions

- $1.$ Define vernier constant.
- $2.$ Write the use of a sliding strip or stem of a vernier caliper.
- $3.$ Explain the method to find zero error in the vernier caliper.
- $4.$ Explain the principle of vernier caliper briefly.

Suggested assessment scheme

(To be filled by teacher)

The given performance indicators should serve as a guideline for assessment regarding the process and product-related marks.

* Marks and percentage weightage for product and process assessment will be decided by the teacher.

2. Determine the diameter of solid ball, wire and thickness of cardboard using a screw gauge.

Practical significance

In industries and laboratories diameter and thickness of objects need to measure with dimensions less than 0.1mm then normal scales and vernier caliper are cannot be used, in such kinds of measurements screw gauge is useful to measure dimensions up to 0.01mm with high precision and accuracy. In this experiment use and the principle of measurement of screw gauge are described.

Relevant theory

Pitch: One complete rotation is given to the screw then the distance travelled by the tip of the screw-on main scale is called the pitch of the screw gauge. It is the distance between two consecutive threads of the screw. The pitch of the screw gauge is equal to 1mm.

Least count of screw gauge: the minimum distance measured by a screw gauge accurately is called the least count of screw gauge.

distance travelled screw The pitch of screw gauge $=$ number of rotation given to screw pitch 1_{mm} Least count = $\frac{r^{2}}{\text{total number of divisions on the circular scale}} = \frac{r^{2}}{100} = 0.01 \text{mm} = 0.001 \text{cm}$

Total reading of diameter = main scale reading + coincide, division of circular scale \times L.C.

Total reading of thickness of cardboard= main scale reading+ coincides division of circular scale \times $(L.C.)$

 $= X + (n \times L.C).$

Corrected reading of diameter = Total reading of diameter- $(±$ zero error)

Corrected reading of thickness of cardboard=Total reading of thickness of cardboard- (± zero error)

Zero error of screw gauge

When the end of the screw and the surface of the stud are in contact with each other, the linear scale (or Main scale) and the circular scale reading should be zero. In case this is not shown, the screw gauge is said to have an error that is said to be zero error. When faces A and B are in contact there is no zero error because zero marks of the linear scale and the circular scale are coinciding with each other as shown in (Fig.1.5). When the reading on the circular scale across the linear scale is more than zero or positive, the screw gauge has positive zero error and value of positive zero error = $0.0 +$ $2(0.002) = (+) 0.004$ cm as shown in (Fig.1.6). When the reading of the circular scale across the linear scale is less than zero or negative and value of negative zero error = $0.0+46$ (0.002) = (-) 0.092cm as shown in $(Fig.1.7)$.

Fig.1.7 negative zero error

Fig.1.5 no zero error

Fig.1.6 positive zero error

Practical outcomes (PrO)

PrO1: Determine the least count and zero error of screw gauge

PrO2: Apply knowledge to find the diameter of ball and wire using a screw gauge

PrO3: Find the thickness of cardboard using a screw gauge

Practical setup (drawing/sketch/circuit diagram/work situation)

Fig.1.8 : Screw gauge A-Anvil or Stud, S-Spindle, N-Lock Nut, M-Frame, H-Barrel, K-Thimble, R-Ratchet screw

Resources required

Precautions

- The screw should move freely without friction. $\mathbf{1}$.
- $2.$ The screw should be moved in the same direction to avoid the back-lash error of the screw.
- $3₁$ Excess rotation should be avoided.

Suggested procedure

- $1.$ Find the distance travelled by the tip of the screw in five complete rotations given on the main scale then divide by 5 of this distance that is equal to the pitch of the screw gauge.
- $2.$ Count number of divisions on circular scale and pitch divided by the total number of divisions on the circular scale that is equal to last count $(L.C.)$ of screw gauge.
- $3.$ Rotate ratchet of screw gauge such that fix end A and movable tip B of screw gauge touch each other and further rotation is not possible of ratchet
- Note down coincide division of circular scale and main scale then find out zero error. $4.$
- 5. Now insert wire between fix end A and the tip of movable screw B then rotate ratchet such that both end A and B touch the wire and no further rotation can make possible
- 6. Then note down the reading of the main scale and coincide, division of the circular scale.
- 7. Keep wire perpendicular to the position of the previous and again note down the reading.
- 8. Repeat steps 5,6 $\&$ 7 and take 4 to 5 readings and then find the mean diameter of the wire.
- 9. To get the corrected value of the diameter of the wire, subtract zero error with (\pm) sign if needed
- 10. Insert cardboard between stud A and tip B, determine the thickness at five different positions.
- 11. Insert the given solid ball between the screw and the stud of the screw gauge and determine diameter at five different positions.
- 12. Find the mean diameter of the solid ball and thickness of cardboard, subtract zero error.

Observations and calculations Observations

To determine pitch: Distance travelled by tip of screw in 5 complete rotations is X

So, pitch = $\frac{X}{5}$ =cm, Total no. of division on the circular scale (N) =

pitch

Least count (L.C.) of screw gauge = $\frac{1}{\text{total number of divisions on the circular scale}}$ =........cm

zero error: reading of zero error = $X \pm n \times L.C. = \pm$ cm

Observation Table: 1. To determine the diameter of the wire

	Reading along one direction				Reading along perpendicular direction				(cm)	
S. N.	Main scale reading $\binom{2}{3}$	of circular $\widehat{\epsilon}$ division scale Coincide	$\pmb{\times}$ \blacksquare Ш scale ϵ Reading of circular ڹ	Total Reading c) cm $\frac{1}{6}$	Main scale reading $\binom{cm}{2}$	of circular Ξ division scale Coincide	\times \mathbf{C} Ш scale $\overline{(\text{cm})}$ of circular ن ٺ Reading	Total Reading c') (cm) \mathbf{a}^+	$(d+d')$ $\overline{2}$ \mathbf{H} Measured diameter	Mean diameter (cm)
	(a)	(b)	(c)	(d)	(a')	(b')	(c')	(d')	(e)	(f)
1.10 5.										

To determine the diameter of the solid ball and the wire 1.

NOTE: Draw the same table again for the wire in the practical notebook

Results and/or interpretation

- Corrected value of diameter of wire $(D1)$ =mean diameter of wire -zero error (with sign) a. $=$cm
- b. Corrected value of diameter of ball (D2) = mean diameter of ball -zero error (with sign) = cm
- Corrected value of thickness of cardboard (t)=mean value of cardboard (zero error) =....cm c_{\cdot}

Conclusions and/or validation

(To be filled by student)

Practical related questions or viva-voce questions

- 1. Write about the term pitch of screw gauge.
- $2.$ Write positions of positive and negative zero error.
- $3.$ Explain the method of minimizing the backlash error of the screw gauge.
- 4. Explain the method to determine the maximum range of the screw gauge.

Suggested assessment scheme

(To be filled by teacher)

The given performance indicators should serve as a guideline for assessment regarding the process and product-related marks.

* Marks and percentage weightage for product and process assessment will be decided by the teacher.

To determine the radius of curvature of a convex and a concave mirror/ $3₁$ surface using a spherometer.

Practical significance

In industries like the oil industry and mechanical engineering, engineers are using spherometer to measure the curvature of different curved surfaces like the surface of pipes, metal plates and spheres. The radius of curvature of curved surfaces can be measured with high precision up to 0.01mm using a spherometer. In civil engineering, a spherometer is useful to measure the curvature of cylinder and lens in surveying. In the petroleum industry spherometer is used to measure the flatness of metal surface and roundness of pipes in the drilling process. In this experiment use and the principle of spherometer are given.

Relevant theory

Pitch: distance travelled by the circular scale in one complete rotation on the main scale.

Least count = $\frac{ue \text{ pucn of the spherometer screw}}{\text{total number of divisions on the circular scale}} = \frac{1 \text{ mm}}{100} = 0.01 \text{ mm} = 0.001 \text{ cm}$ The formula for the radius of curvature of the curved surface (R) = $\frac{a^2}{6h} + \frac{h}{2}$

Here 'a' is the average distance between two legs of the spherometer and 'h' is the average height of a curved surface from a plane surface which is known as sagitta.

Practical outcomes (PrO)

- PrO1: Find the least count and zero error of spherometer
- PrO2: Use spherometer to find the sagitta of convex and concave mirror/surface
- PrO3: Find the radius of curvature of convex and concave mirror/surface using spherometer

Practical setup (drawing/sketch/circuit diagram/work situation)

Fig. 1.9: Schematic diagram of spherometer $A, B, C = legs$, $D = circular scale$, $O = central leg$, $p = vertical scale(or main scale)$

Resources required

Precautions

- $1.$ The screw should move freely without friction.
- The screw should be moved in the same direction to avoid the back-lash error of the screw. $\overline{2}$.
- \mathcal{R} Excess rotation should be avoided.

Suggested procedure

- Raise the central screw of the spherometer and press the spherometer gently upon the practical 1. notebook to get pricks of the three legs. Mark these pricks as A, B and C.
- $2.$ Measure the distances between the pricks (points) by joining the points to form a triangle ABC.
- \mathcal{R} Note these distances (AB, BC, AC) on notebook and take their mean that is the value of 'a'.
- Determine the pitch and the least count of the spherometer and record it stepwise. 4.
- 5. Place the spherometer on the convex surface and gently, turn the screw downwards till the tip of the central screw just touches the convex surface (The tip of the screw will just touch its image in the convex glass surface) and all three legs of the spherometer are also remain in contact with that convex surface.
- 6. Note the reading of the main scale (or vertical scale) and also read the divisions n of the circular scale which is in line with the vertical scale then find the total reading = Main Scale reading + $n \times L.C.$ While taking a reading on convex surface main scale reading is measured above the zero of that vertical scale.
- Repeat steps $5 \& 6$ to take readings of the 7. spherometer at three different places on the convex surface. The mean of readings taken on the convex surface of the spherometer is $\mathbf{\hat{h}}_2$.
- 8. Repeat same steps $5 \& 6$ to take readings on a concave surface, in the case of concave surface main scale reading measured below the zero of that vertical scale. The mean of the readings taken at three different places on a concave surface is \mathcal{R}_{3} .

Fig. 1.10: Measurement of sagitta 'h'

9. Now take spherometer reading on a plane glass plate by repeating steps $5 \& 6$ at three different places on it and determine the mean of readings on the plane surface that is h' .

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10. Calculate $(h_2 - h_1)$ = h the value of sagitta (h) (or height of convex surface from the plane surface). Similarly $(h_3 - h_1) = h'$ the value of sagitta (h') (or depth of concave surface from the plane surface). Find R and R' radius of curvature for convex and concave surfaces.

Observations and calculations

To find the average distance between two lags of the spherometer In shown (Fig 1.11) of triangle ABC marked by the legs of spherometer $AB = a_1 = \dots \dots \text{cm}, BC = a_2 = \dots \dots \text{cm}, CA = a_3 = \dots \dots \text{cm}$ So, the average distance between lags of the spherometer

$$
a = \frac{a_1 + a_2 + a_3}{3} = \dots \dots \text{cm}
$$

For the Pitch: distance travelled by central screw in 5 complete rotations of circular scale is X

Pitch = $\frac{X}{5}$ =.......cm, Total no. of division on the circular scale (N) =

pitch

Least count of spherometer (LC) = $\frac{1}{\text{total number of division on the circular scale}}$ $=$cm

Spherometer readings:

Results and/or interpretation

height of convex surface from plane surface (or sagitta) $h = h₂ - h₁ =$ cm depth of concave surface from plane surface (or sagitta) $h' = h_3 - h_1 = \dots \dots$ cm

radius of curvature of convex surface
$$
R = \frac{a^2}{6h} + \frac{h}{2} = \dots
$$
 cm

the radius of curvature of concave surface R' = $\frac{a}{6h'} + \frac{b}{2}$ =.......cm

$$
\frac{1}{6h} + \frac{1}{2} = \dots \dots c
$$

$$
P' = \frac{a^2}{2} + \frac{h'}{2}
$$

Conclusions and/or validation

(To be filled by student)

Practical related questions or viva-voce questions

- Explain zero correction of spherometer. 1.
- $2.$ Write the relation between focal length and radius of curvature of the convex mirror.
- $\overline{3}$. Give method to minimize of backlash error of the spherometer.
- $\overline{4}$. Define the least count of an instrument.

Suggested assessment scheme

(To be filled by teacher)

The given performance indicators should serve as a guideline for assessment regarding the process and product-related marks.

* Marks and percentage weightage for product and process assessment will be decided by the teacher.

KNOW MORE

Units of length for astronomical objects outside the solar system are parsec, light years or astronomical units (AU). Masses of molecules or atoms are measured in atomic mass unit(amu) while mass of astronomical objects like galaxies or blackholes is measured in solar mass. Informal unit of time that measures 10 nanoseconds or 10^{-8} seconds is known as 1 shake. The area of application of this unit of time is in nuclear reactions to express the timing of events like neutron reactions.

Why accuracy and reliability of measurements are so important?

A lack of knowledge, accuracy and miscalculations can cause disaster. One of such examples is the great Kersten Blunder. His mistake was in conversion from millimetres to inches that cost the government over 2 billion dollars. Instead of setting the value to 25.4, he set 24.5 which resulted in the Vigor space probe missing the planet Venus and was lost in space consequently.

Innovative Practical /Projects/ Activities

- (1) Measure the object using Vernier Caliper of different least counts.
- (2) Measure the object using a micrometer screw gauge of different least counts.
- (3) Measure the circular object using a spherometer of different least counts.
- (4) List down various measuring instrument in your core programs.

REFERENCES AND SUGGESTED READINGS

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Suggested Learning Resources for Practical

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Force and Motion

UNIT SPECIFICS

This unit focusses on the following aspects of basic physics:

- Scalar and vector physical quantities and their examples
- Mathematical operations on vector quantities and applications of resolution of a vector
- Triangle and Parallelogram law (Statement only)
- Conservation of linear momentum and its applications
- Physical quantities related to circular motion of an object
- Relation between linear and angular velocity, linear acceleration and angular acceleration
- Centripetal and Centrifugal forces with examples
- Expression and applications such as banking of roads and bending of cyclist.

RATIONALE

Any measurable quantity is either a scalar or a vector. A scalar quantity can be described by its magnitude only and no need of direction. A vector quantity is described by magnitude and direction both and it must obey the law of vector addition. Triangle law and parallelogram law are used for vector addition. The product of two vectors and the resolution of vectors are explained with examples. Law of conservation of angular momentum and impulse are derived, applications like the recoil of gun and rocket are described. Angular displacement, angular velocity, angular acceleration and their relations with linear quantities are derived. Centripetal and centrifugal forces with applications like banking of roads, bending of cyclist are also explained in the text.

PRE-REQUISITES

Physics – basics of units and measurements of physical quantities

Mathematics – basics of linear algebra

Other – basic knowledge of computer

UNIT OUTCOMES

- U2-O1: Distinguish between vector and scalar quantities. Apply vector addition law, subtraction and product of vectors. Use resolution of vectors on inclined plain and lawn roller.
- U2-O2: Establish the relationship of force with momentum and impulse. Apply conservation of linear momentum in rocket propulsion and recoil of the gun.
- U2-O3: Define angular motion and associated physical quantities. Identify centripetal and centrifugal forces in living examples and able to use them in banking of roads and bending of cyclist.

Mapping of the unit outcomes with the course outcomes

$2₁$ **SCALAR AND VECTOR QUANTITIES**

Interesting Facts

The vector in a plane (like a plain sheet of paper or a plain sheet or in two-dimensional space) is described by the adding of two linearly independent vectors. Vectors in 3-dimensional space can be written as the sum of three vector components. Vectors are the mathematical tool that can be used to solve equations of physics and make the calculation easier.

2.1.1 Scalar and Vector Quantities

Certain physical quantities are described completely by only their magnitude such quantities are known as scalars. These quantities are added only by the rules of algebra. Mass, volume, density and temperature are some of the examples of scalar quantities.

Description of some physical quantities needs magnitude, as well as direction and these quantities, are added by the law of vector addition, such quantities are called vectors. Velocity, acceleration, linear momentum and force are some of the examples of vector quantities.

In some cases, a physical quantity has magnitude as well as direction but can not be added by the law of vector addition, that quantity is not a vector. For example: electric current has both magnitude and direction in a wire but current can not be added by the law of vector addition, thus, the current is a scalar quantity.

Representation of vector Α.

Geometrically, a vector is represented by a straight line with an arrowhead, the length of the line is taken proportional to the magnitude of that quantity and the arrowhead shows the direction of that vector.

In (Fig. 2.1), the vector \vec{A} is represented by \overline{PQ} , length PQ gives the magnitude or modulus of the vector A and the arrowhead shows the direction of the vector. Vectors are written by putting an arrow over the symbols like \overline{PQ} , \overline{AB} etc. Sometimes we use single-letter symbols as $\vec{r}, \vec{f}, \vec{v}$ etc. In some books, vectors are represented by bold letters such as PQ, AB, F, V etc.

The magnitude of the vector is denoted by $|\overrightarrow{A}|$ modulus or by letter A (without bold).

Types of vector В.

- 1. **Null or zero vector:** When the initial and final points (or terminal points) of a vector coincide, then the magnitude of the vector is zero that vector is called a null vector.
- 2. Unit vector: If the magnitude of a vector is unity, then it is called a unit vector. A unit vector

of vector
$$
\vec{A}
$$
 is denoted as $\hat{A} = \frac{A}{|\vec{A}|} = \frac{\text{vector}}{\text{magnitude of vector}}$

A is called A cap.

3. Equal vector: Two vectors are said to be equal if they represent the same physical quantity with the same magnitude and direction.

 $\vec{A} = \vec{B}$

4. Negative vector: When two vectors have equal magnitude but opposite in direction to each other are called negative or opposite vector which can be written as $\vec{A} = -\vec{B}$.

Types of vector depending on the initial point

- a. Free vector: If the initial point of a vector can be displaced to any point in the space, then it is called a free vector.
- b. Sliding vector: When the initial point of a vector can be displaced only along the line of its action then it is called a sliding vector.
- **Bound vector:** If the initial point of a vector is fixed in the space and cannot be changed then $c.$ it is called a bound vector.
- d. Polar vector: If the direction of a vector independent of the reference frame, then it is called a polar vector. The direction of polar vector shown by their description like velocity, acceleration, force vector.
- e. Axial vector: If a vector is represented rotational effect and is acted along the axis of rotation then it is called an axial vector. The direction of the axial vector depends on the handedness of the reference frame.

Torque $\vec{\tau} = \vec{r} \times \vec{F}$ and angular momentum $\vec{J} = \vec{r} \times \vec{p}$ are some examples of axial vectors.

Coplanar vector: If two vectors lie in the same plane then they are called coplanar vectors.

$2.1.2$ **Addition and Subtraction of Vectors**

Addition of two vectors Α.

Triangle law: when two vectors \vec{a} and \vec{b} to be added then put the tail of \vec{b} on the head of \vec{a} and the vector joining the tail of the first vector \vec{a} with the head of the second vector \vec{b} is the vector sum of these two vectors \vec{a} and \vec{b} . This rule is known as the triangle law of the addition of two vectors.

Parallelogram law: If two vectors \vec{a} and \vec{b} are represented by two adjacent sides of a parallelogram with their common tails then the diagonal drawn through the intersection of the two vectors gives the vector sum of \vec{a} and \vec{b} .

$$
\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} \qquad \overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AC}
$$

The magnitude of $(\vec{a} + \vec{b})$ is given as $|\vec{a} + \vec{b}| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$ where θ is a smaller angle between the vector \vec{a} and \vec{b} .

If $(\vec{a} + \vec{b})$ is making an angle α with vector \vec{a} then, tan $\alpha = \frac{bsin\theta}{a + b\cos\theta}$

В. **Subtraction of two vectors**

Suppose \vec{a} and \vec{b} are two vectors then subtraction of \vec{a} and \vec{b} can be defined as a sum of the vector \vec{a} and the opposite vector of $\vec{b} \cdot (-\vec{b})$ is an opposite vector of vector \vec{b} .

 $\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

As an example, we can take

 $\vec{b} - \vec{a} = \vec{b} + (-\vec{a})$ and $(\vec{b} - \vec{a})$ is in the opposite direction of $(\vec{a} - \vec{b})$.

2.1.3 Resolution of a Vector

Let a vector $\vec{A} = \vec{OP}$ in the x-y plane is drawn from the origin O to point P as shown in (Fig. 2.5)

Let the vector \vec{A} makes an angle α with X-axis and $\beta = (90 - \alpha)$ with Y-axis, then PM and PN are perpendiculars drawn from point P to X and Y axes respectively.

OM = projection of vector \overrightarrow{OP} on X-axis = component of vector \overrightarrow{OP} along X-axis and ON = projection of a vector \overrightarrow{OP} on Y-axis = component of a vector \overrightarrow{OP} along Y-axis By triangle rule of vector addition

$$
\vec{A} = \vec{OP} = \vec{OM} + \vec{MP} = \vec{OM} + \vec{ON}
$$
 (here $\vec{MP} = \vec{ON}$ opposite sides of a rectangle)
\nIn $\triangle OMP$ $\cos \alpha = \frac{OM}{OP}$ or $OM = OP$ $\cos \alpha = A \cos \alpha$
\nIn $\triangle ONP$ $\cos \beta = \frac{ON}{OP}$ or $ON = OP \cos \beta = A \cos \beta$
\nIf \vec{i} and \vec{j} are unit vectors along the X and Y axes respectively
\nThen, we get $\vec{OM} = (A\cos\alpha)\vec{i}$ and $\vec{ON} = (A\cos\beta)\vec{j}$
\nThus, vector $\vec{A} = A_{\vec{x}}\vec{i} + A_{\vec{y}}\vec{j} = (A\cos\alpha)\vec{i} + (A\cos\beta)\vec{j}$
\n $|\vec{A}|^2 = OP^2 = OM^2 + MP^2 = A_{\vec{x}}^2 + A_{\vec{y}}^2$
\nSo, the magnitude of vector $|\vec{A}| = \sqrt{A_{\vec{x}}^2 + A_{\vec{y}}^2}$ and $\tan \alpha = \frac{A_{\vec{y}}}{A_{\vec{x}}}$
\nSimilarly, we can resolve a vector \vec{A} in three-dimensional space along X, Y and Z axes.
\nIf vector \vec{A} makes angle α , β and γ with three axes X, Y and Z respectively, we get
\n $\vec{A} = (A \cos \alpha)\vec{i} + (A \cos \beta)\vec{j} + (A \cos \gamma)\vec{k}$
\nhere, \vec{i} , \vec{j} , \vec{k} are the unit vectors along the X, Y, and Z axes respectively.
\nLet $\vec{A} = A_{\vec{x}}\vec{i} + A_{\vec{y}}\vec{j} + A_{\vec{z}}\vec{k}$, $\vec{B} = B_{\vec{x}}\vec{i} + B_{\vec{y}}\vec{j} + B_{\vec{z}}\vec{k}$
\n $\vec{A} \pm \vec{B} = (A_{\vec{x}} \pm B_{\vec{x}})\vec{i} + (A_{\vec{y}} \pm B_{\vec{y}})\vec{j} + (A_{\vec{z}} \pm B_{\vec{z}})\vec$

$2.1.4$ **Applications of a Vector**

Α. **Inclined plane**

We can demonstrate a force vector on the inclined plane. Let a block of mass 'm' is kept on an inclined plane which makes an angle θ with the horizontal direction.

The weight of block $W = mg$ can be resolved into two components, one is parallel to the inclined plane W_{\parallel} and the other is perpendicular to the inclined plane W_{\perp} .

 $\cos\theta = \frac{AB}{AC} = \frac{W_{\perp}}{W}$ or $W_{\perp} = W \cos\theta = N$ (normal reaction) is a vertical component

B. Lawn roller

Let a lawn roller has actual weight W. This lawn roller is pushed or pulled by a person over a plane lawn.

In case of pushing: the lawn roller is pushed by a person with external force \vec{F} making an angle θ with the horizontal direction. This force \vec{F} can be resolved in two normal components at point O. Here, the horizontal component of this force \vec{F} is equal to F cos θ and the direction of this force component is along with OC. The vertical component of force \vec{F} is equal to Fsin θ and the direction of this component is along with OA. Due to the horizontal component Fcos θ roller moves in the forward direction. If the normal reaction on the roller is N, then,

 $N = W + F \sin \theta$

thus, the apparent weight of the roller increases so, it is difficult to push the roller.

In case of pulling: Now, this lawn roller is pulled by a person with the same external force \vec{F} making an angle θ with the horizontal direction. This force \vec{F} can be resolved again in two normal components at point O. Here, the horizontal component of this force \vec{F} is equal to Fcos θ and the direction of this force component is along with OC. But the vertical component of this force \vec{F} is equal to Fsin θ and it is acting along with OD. If the normal reaction is N' then,

 $N' = W - F \sin \theta$

thus, the apparent weight of the roller decreases so, it is easier to pull the roller.

$2.1.5$ **Scalar and Vector Product of two Vectors**

When a vector \vec{A} is multiplied by a number K we get $K \vec{A}$ which is a vector in the same direction but magnitude K times of magnitude of the vector \vec{A} . If vector \vec{A} is multiplied by -K then we get $(K \overline{A})$ a vector in the opposite direction but magnitude K times of the vector \overline{A} .

Scalar or dot product Α.

The dot product of two vectors \vec{A} and \vec{B} is given as

 $\vec{A} \cdot \vec{B} = AB \cos \theta = |\vec{A}| |\vec{B}| \cos \theta$

Here, A and B are the magnitudes of vectors \vec{A} and \vec{B} respectively

and θ is the angle between them.

If the angle between the vector \vec{A} and \vec{B} is 90° or \vec{A} and \vec{B} are mutually perpendicular then dot product $\vec{A} \cdot \vec{B} = AB \cos \theta = 0$ as $(\cos 90^\circ = 0)$

The dot product does not depend on the order of vectors so, it follows commutative law and also follows distributive law.

 $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ (Commutative law)

 $\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$ (Distributive law)

Example: Two vectors of magnitude $A=10$ and $B=20$ and angle between them is 120 $^{\circ}$. Determine the dot product of these two vectors.

 $\vec{A} \cdot \vec{B} = AB \cos \theta = 10 \times 20 \times \cos 120^\circ = -100$ units

The dot product of two vectors is a scalar quantity.

The dot product of two vectors can be found if vectors are given in the form of components along the coordinate axes. Let two vectors \vec{A} and \vec{B} are given in terms of unit vectors \vec{i} , \vec{j} and \vec{k}

$$
\vec{A} = A_{x} \vec{i} + A_{y} \vec{j} + A_{z} \vec{k}, \quad \vec{B} = B_{x} \vec{i} + B_{y} \vec{j} + B_{z} \vec{k}
$$
\nThen, $\vec{A} \cdot \vec{B} = (A_{x} \vec{i} + A_{y} \vec{j} + A_{z} \vec{k}) \cdot (B_{x} \vec{i} + B_{y} \vec{j} + B_{x} \vec{k})$ \n
$$
= A_{x} B_{x} (\vec{i} \cdot \vec{i}) + A_{x} B_{y} (\vec{i} \cdot \vec{j}) + A_{x} B_{z} (\vec{i} \cdot \vec{k}) + A_{y} B_{z} (\vec{j} \cdot \vec{i}) + A_{y} B_{y} (\vec{j} \cdot \vec{j}) + A_{y} B_{z} (\vec{j} \cdot \vec{k}) + A_{z} B_{x} (\vec{k} \cdot \vec{i}) + A_{z} B_{y} (\vec{k} \cdot \vec{j}) + A_{z} B_{z} (\vec{k} \cdot \vec{k})
$$
\n
$$
\vec{A} \cdot \vec{B} = A_{x} B_{x} + A_{y} B_{y} + A_{z} B_{z}
$$

Here, we can write the scalar product of unit vectors along X , Y and Z axes.

 $(\vec{i}, \vec{i}) = (\vec{j}, \vec{j}) = (\vec{k}, \vec{k}) = 1 \cdot 1 \cos 0^\circ = 1$ and $(\vec{i}, \vec{j}) = (\vec{j}, \vec{k}) = (\vec{k}, \vec{i}) = 1 \cdot 1 \cos 90^\circ = 0$

\boldsymbol{B} . **Vector Product or Cross Product of two Vectors**

Vector product or cross product of two vectors \vec{A} and \vec{B} is denoted by $\vec{A} \times \vec{B}$ and it is also a vector itself. Then magnitude of this vector given as $|\vec{A} \times \vec{B}| = AB \sin \theta$

Here A and B are the magnitudes of vectors \vec{A} and \vec{B} respectively and θ is the angle between these two vectors.

The direction of the vector $\vec{A} \times \vec{B}$ is perpendicular to both vector \vec{A} and \vec{B} . Thus, $\vec{A} \times \vec{B}$ is perpendicular to the plane containing vector \vec{A} and \vec{B} . The direction of $(\vec{A} \times \vec{B})$ can be found with the help of the right-hand rule. According to this rule, both vector \vec{A} and \vec{B} should be placed tail to tail and to find out the direction of $A \times B$ point the fingers of right hand along the vector A with palm facing the vector \vec{B} . When the fingers are curled towards \vec{B} , the thumb of right-hand points to the direction of $\overrightarrow{A} \times \overrightarrow{B}$.

Note: $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ so, cross-product is not commutative. Because $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$ Distributive law can be applied to cross product $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$ Associative law cannot be applied to cross product $\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$ The cross product of unit vectors is given below $\vec{i} \times \vec{j} = (1.1 \sin 90^\circ) \vec{k} = \vec{k} = -\vec{j} \times \vec{i}$ $\vec{i} \times \vec{k} = -k \times \vec{j} = \vec{i}$, $\vec{k} \times \vec{i} = -\vec{i} \times \vec{k} = \vec{j}$ and $\vec{i} \times \vec{i} = (1.1 \sin 0^\circ) = 0$ similarly $\vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$ If vector \vec{A} and \vec{B} are given in terms of component then, Fig. 2.12 cross product of two vectors can also be expressed in determinant form as $\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \end{bmatrix}$. .

X

$$
\vec{A} \times \vec{B} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \vec{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \vec{j} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \vec{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}
$$

$$
= (A_y B_z - A_z B_y) \vec{i} + (A_z B_x - A_x B_z) \vec{j} + (A_x B_y - A_y B_x) \vec{k}
$$

Applications (Real-life/Industrial)

1. When a car is moving, the tyre has initial velocity, acceleration, the friction force between the tyre and the road. The gravitational reaction also applied to the tyre. So, all these vectors are used in the description of the motion of a car.

- 2. While a boatman crosses the river to reach straight on the other bank of the river then he should keep in mind both velocity of the river and the boat.
- 3. In physics and engineering vectors are used in the description of electromagnetic and gravitational field vector.

Case-Study (Environmental/Sustainability/Social/Ethical Issues)

Torque is given by vector product of force arm and external force applied. In the case of a door, the handle is attached near the end of the door so that the length of the force arm longer due to that less force is required to shut or open the door. The cross-product of two vectors is also a vector. Thus, the torque has a direction.

Torque = force arm \times external force = $\vec{r} \times \vec{F}$

The door is rotated about the vertical line passes through the hinges along the direction of the torque and perpendicular to the plane of \vec{r} and \vec{F} .

Create Inquisitiveness and Curiosity

Discuss two examples of those quantities having both magnitude and direction but quantities are not the vectors. Explain power is a scalar, not a vector quantity. Discuss, a scalar quantity cannot be added to a vector quantity. Explain magnitude of a vector can not be negative. If the sum of three vectors is zero explain that geometrical condition.

Solved problems

Problem-1: Resolve in horizontal and vertical components of a force $F = 20N$ which is making an angle 60° with the horizontal.

Solution: Horizontal component
$$
F_x = F \cos 60^\circ = 20 \times \frac{1}{2} = 10N
$$
,
\nVertical component $F_y = F \cos 30^\circ = 20 \times \frac{\sqrt{3}}{2} = 10 \sqrt{3}N$
\n**Problem-2:** Find the magnitude of $\vec{A} - 2\vec{B}$, if $\vec{A} = 2\vec{i} - \vec{j} + 2\vec{k}$ and $\vec{B} = 3\vec{i} + \vec{j} + 4\vec{k}$
\n**Solution:** $\vec{A} - 2\vec{B} = 2\vec{i} - \vec{j} + 2\vec{k} - 2(3\vec{i} + \vec{j} + 4\vec{k}) = -4\vec{i} - 3\vec{j} - 6\vec{k}$
\nSo, the magnitude of $|\vec{A} - 2\vec{B}| = \sqrt{(4)^2 + (-3)^2 + (-5)^2} = \sqrt{16 + 9 + 36} = \sqrt{61}$
\n**Problem-3:** If $\vec{A} = 4\vec{i} - 6\vec{j} + 8\vec{k}$ and $\vec{B} = 2\vec{i} + 4\vec{j} - 6\vec{k}$ are two vectors. Determine $\vec{A} \times \vec{B}$.
\n**Solution:** $\vec{A} \times \vec{B} = (4\vec{i} - 6\vec{j} + 8\vec{k}) \times (2\vec{i} + 4\vec{j} - 6\vec{k}) = 4 \times 2(\vec{i} \times \vec{i}) + 4 \times 4(\vec{i} \times \vec{j}) + 4 \times (-6)(\vec{i} \times \vec{k}) + (-6) \times (2)(\vec{j} \times \vec{i}) + (-6) \times 4(\vec{j} \times \vec{j}) + (-6)(-6)(\vec{j} \times \vec{k}) + 8 \times 2(\vec{k} \times \vec{i}) + 8 \times 4(\vec{k} \times \vec{j}) + 8 \times (-6)(\vec{k} \times \vec{k}) = 0 + 16\vec{k} - 24(-\vec{j}) - 12(-\vec{k}) - 0 + 36\vec{i} + 16\vec{j} + 32(-\vec{i}) - 0$
\n $\vec{A} \times \vec{B} = 4\vec{i} + 40\vec{j} + 28\vec{k}$
\n**Problem-4:**

FORCE AND MOMENTUM 2.2

Interesting Facts

The force said to be exerted when a person kicks the football, it comes to rest after some time due to force exerted on it in the opposite direction of motion. The total motion of a body is defined by linear momentum. If the net force is zero then the total linear momentum of the system remains constant. These facts will be studied in the content.

$2.2.1$ **Force**

Force is a push or pulls which can change or tends to change the state of rest or uniform motion of an object while exerts on it. Force may change the speed, the direction of motion of a body and may change the size and shape of a body. S.I unit of force is the newton (N). Force is a vector quantity and net force on a body can be found by rules of vector addition. There is four natural forces (1) Gravitational (2) Electromagnetic (3) Nuclear (4) Weak force. Force is classified into two categories (a) Contact force: when the source of a force remains in contact with an object then it is called contact force like muscular force, tension (b) distant force: in this type of force, source of a force needs not to come in contact with an object like gravitation and electrostatic force.

Linear momentum: it is the total motion of the body. The linear momentum of a body is equal to the product of the mass (m) and its velocity \vec{v} . Linear momentum is given as $\vec{p} = m\vec{v} \vec{p}$ is a vector quantity and its direction is along the direction of the velocity \vec{v} of the body.

Newton's second law of motion: change in momentum of a body is proportional to the net external force exerted on that body.

If the externally applied force is \vec{F} then

$$
\vec{F} \propto \left(\frac{\vec{p}_2 - \vec{p}_1}{\Delta t}\right) \text{ or } \vec{F} = k \frac{m(\vec{v}_2 - \vec{v}_1)}{\Delta t} \text{ or } \vec{F} = m\vec{a},
$$

k is constant and k = 1, (here acceleration $\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$)

 $\vec{p}_1 = m\vec{v}_1$ and $\vec{p}_2 = m\vec{v}_2$ are the initial and final linear momentum of the body respetively.

Newton's third law of motion: when a body A exerts a force on B then, at the same instant equal force is exerted by B on A but in opposite direction. It is also called the action and reaction law. Here, either force may be action and the other are the reaction. Action and reaction are exerted on two different bodies. $\vec{F}_{AB} = -\vec{F}_{BA}$

Law of Conservation of Linear Momentum $2.2.2$

If the net force exerted on a body (or system) is zero then,

 $\vec{F} = \frac{\vec{p}_2 - \vec{p}_1}{\Delta t} = 0$ (by newton's second law of motion)
or $\vec{p}_1 = \vec{p}_2$

 \vec{p} = constant. Thus, if no force is applied to the system, then the total linear momentum of the system remains constant or conserved. This fact is called the law of conservation of linear momentum.

Application of Conservation of Linear Momentum $2.2.3$

Recoil of gun А.

Let a bullet of mass 'm' is fired from a gun of mass 'M'. If the force exerted on the bullet by the gun is \vec{F} , then equal and opposite force $(-\vec{F})$ is exerted by bullet on the gun according to newton's third law. The sum of internal forces (gun + bullet) is zero. There is no external force applied so the net force on the system $(gun + bullet)$ is zero.

Thus, the initial total momentum of (gun + bullet) before the bullet is fired = final total momentum of (gun + bullet) after bullet fired

 $0 = m\vec{v} + M\vec{V}$

Initially both bullet and gun are at rest so total linear momentum is zero. Here, \vec{v} and \vec{V} are the velocities of bullet and gun respectively, just after bullet fired.

 $M\vec{V} = -m\vec{v}$ or Recoil velocity of the gun $\vec{V} = -m\vec{v}/M$

The velocity of the gun with which it moves backward is called recoil velocity and the negative sign shows direction opposite to bullet.

Rocket B.

Rocket propulsion is an example of the conservation of linear momentum. The rocket consists of a combustion chamber filled with liquid fuel in which the fuel is burnt and heat rises pressure inside the chamber. Therefore, hot gases exhaust through the nozzle in form of a jet with very high speed in the opposite direction of motion of the rocket. There is no external force exerted on the system (rocket + gases), ignore force due to gravity. So, the linear momentum of (rocket $+$ gases) remains conserved. Hot gases exhaust in a backward direction and the rocket moves in the forward direction with equal linear momentum.

Suppose at the time $\Delta \mathfrak{t}$ the mass of rocket with fuel is 'm' and initial velocity of the rocket is \vec{v} . So, the initial linear momentum of (rocket + fuel) $\vec{p} = m\vec{v}$ $...(1)$

After time interval Δt velocity of the rocket increases by $\Delta \vec{v}$ and mass decreases by Δm . Where Am is the mass of gases which exhaust after fuel burnt. Let the velocity of gases with respect to the rocket is \vec{u} . Then the velocity of gas with respect to the rocket is given as $\vec{v}_{GR} = \vec{v}_G - \vec{v}_R$

so, the velocity of the gas $\vec{v}_G = \vec{v}_R + \vec{v}_{GR} = (\vec{v} + \Delta \vec{v} - \vec{u})$

(here $\vec{v}_{GR} = -\vec{u}$ gases exhaust in the opposite direction of motion of the rocket)

Final momentum of (rocket + gases) = momentum of rocket + momentum of gases

$$
\vec{p}' = (m - \Delta m)(\vec{v} + \Delta \vec{v}) + \Delta m(\vec{v} + \Delta \vec{v} - \vec{u})
$$

=
$$
m\vec{v} + m(\Delta \vec{v}) - (\Delta m)\vec{v} - (\Delta m)(\Delta \vec{v}) + (\Delta m)\vec{v} + (\Delta m)(\Delta \vec{v}) - (\Delta m)\vec{u}
$$

\n $\vec{p}' = m\vec{v} + m(\Delta \vec{v}) - (\Delta m)\vec{u}$...(2)

by the law of conservation of linear momentum, $\vec{p} = \vec{p}'$

$$
m\vec{v} = m\vec{v} + m(\Delta \vec{v}) - (\Delta m)\vec{u}
$$

m($\Delta \vec{v}$) = (Δm) \vec{u} ...(3)

Divide both side of the equation by time interval Δt , thus, $m \frac{(\Delta \vec{v})}{\Delta t} = \frac{\Delta m}{\Delta t} \vec{u}$

$$
\vec{F} = m \frac{(\Delta \vec{v})}{\Delta t} = \frac{\Delta m}{\Delta t} \vec{u}
$$

The thrust of rocket = mass \times Acceleration of rocket = velocity of gasses \times rate of mass decreases. Acceleration of rocket

$$
\vec{a} = \frac{\vec{F}}{m} = \frac{\vec{u}}{m} \left(\frac{\Delta m}{\Delta t} \right)
$$

$$
\Delta \vec{v} = \vec{u} \left(\frac{\Delta m}{m} \right) \text{ from equation (3)}
$$

Equation of velocity of rocket:

$$
d\vec{v} = \vec{u} \left(\frac{dm}{m} \right) \text{ (for small values } \Delta \vec{v} \to d\vec{v} \text{ , } \Delta m \to dm \text{ and } \Delta t \to dt \text{)}
$$

If at t = 0 mass of the rocket is m_i and velocity is \vec{v}_i , the final mass of the rocket is m_f and final velocity is \vec{v}_f

Then, by integration

$$
\int_{v_i}^{v_f} d\vec{v} = \vec{u} \int_{m_i}^{m_f} \frac{dm}{m}
$$
\n
$$
[\vec{v}]_{v_i}^{v_f} = \vec{u} [\log m]_{m_i}^{m_f} = \vec{u} [\log m_f - \log m_i] = -\vec{u} \log \frac{m_i}{m_f}
$$
\n
$$
\vec{v}_f - \vec{v}_i = -\vec{u} \log \frac{m_i}{m_f}
$$
\n
$$
\vec{v}_f = \vec{v}_i + \vec{v}_r \log \frac{m_i}{m_f} \quad (\vec{v}_r = -\vec{u} \text{ the relative velocity of gases with respect to the rocket})
$$

If the initial velocity of the rocket is zero($\vec{v}_i = 0$), the final velocity of the rocket $\vec{v}_f = \vec{v}_r \log \frac{m_i}{m_f}$

Impulse and its Applications $2.2.4$

Impulse is equal to change in linear momentum of a body. In another form, the impulse is also defined as the product of force and time for which external force acts on the body. By newton's second law of motion,

$$
\vec{F} = \frac{\vec{p}_2 - \vec{p}_1}{\Delta t} \text{ or } \vec{F} \Delta t = \vec{p}_2 - \vec{p}_1
$$

Impulse $\vec{l} = \vec{F}\Delta t = \vec{p}_2 - \vec{p}_1$ here $\Delta t = (t_2 - t_1)$ is time interval for which force acts

Fig. 2.15

Impulse-momentum theorem: The impulse of a force applied on a body is equal to the change in linear momentum of that body. Impulse is a vector; its SI unit is $(N-s)$ and dimensions are MLT⁻¹.

Applications of impulse

- (1) When a cricketer catches the ball, he pulls his hands a little backwards. Doing so takes some more time which further reduces the momentum of the ball. As time 't' increases relatively smaller force F is applied by the ball so, the hands remain safe or do not injure.
- (2) It is easier to catch a tennis ball than a cricket ball. A cricket ball has a larger mass than a tennis ball. If both balls are moving at the same speed, a greater change in momentum is involved in the case of a cricket ball hence a larger force is applied by a cricket ball.

Applications (Real-life/Industrial)

- 1. When a bus starts suddenly then the passenger feels jerk backwards.
- 2. Passengers feel jerk in the forward direction when suddenly breaks are applied on the bus.
- 3. When a man walks on the ground, he pushes the ground backwards and the ground pushes the man forward. According to Newton's third law of motion, a man can walk.
- 4. In engineering, rocket propulsion is an example of the combined effect of conservation of linear momentum and newton's third law of linear motion.

Case-Study (Environmental/sustainability/social/ethical issues)

Impulse is a product of force and time for which it acts. Impulse $\vec{I} = \vec{F} \times \Delta t = \vec{p}_f - \vec{p}_i$

Two balls of mass $m_1 = 20g$ and $m_2 = 50g$ drop from same height and ball of mass 50g rebound but the ball of 20g is not rebound. In this example impulse of second ball = $mv_f - (-mv_i)$ will be more than impulse of first ball = $(0 - (-mv_i))$

Create Inquisitiveness and Curiosity

- 1. Discuss that in the non-inertial frame even no external force is applied on the body but acceleration of the body does not zero.
- 2. Explain that in the law of conservation of linear momentum even the total momentum of the system $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots$ = constant or remains the same but the linear momentum of individual bodies \vec{p}_1 , \vec{p}_2 , \vec{p}_3 may be changed.

Solved Problems

Problem-1: A block of mass 20kg is suspended with string as shown in (Fig. 2.15). Find the tension in the string. $(g = 10m/s^2)$ **Solution:** Block is at rest so net force is zero, thus,

 $T - mg = 0$ or $T = mg = 20 \times 10 = 200N$

Problem-2: A bullet of mass 40g is fired from the gun with an initial

velocity of 200m/s. If the mass of the gun is 5k, determine the recoil velocity of the gun.

Solution: Recoil velocity of the gun $\overline{V} = -m\overline{v}/M$ here, mass of bullet m = $40g = 40 \times 10^{-3}kg$

(Velocity of bullet $\vec{v} = 200$ m/s and mass of gun M = 5kg)

Recoil velocity of the gun $\vec{V} = -\frac{40 \times 10^{-3} \times 200}{5} = -\frac{8}{5} = -1.6 \text{m/s}$

The recoil velocity of a gun is in the opposite direction of the bullet fired.

Problem-3: The rate of fuel burn of a rocket is 2×10^4 kg/s and the exhaust velocity of gases in form of the jet is 3×10^3 m/s. Find the thrust of the rocket.

Solution: Here, exhaust velocity of gases $v_r = 3 \times 10^3$ and loss of mass per sec in combustion Λ

$$
\frac{\Delta m}{\Delta t} = 2 \times 10^4 \,\mathrm{kg/s}
$$

The thrust of the rocket F = $v_r \frac{\Delta m}{\Delta t} = 3 \times 10^3 \times 2 \times 10^4 = 6 \times 10^7 N$

Problem-4: A rubber ball of mass 300g is dropped from the window of a building. It strikes the footpath (or sidewalk) below at 40m/s and rebounds up with 30m/s. Find the impulse due to collision with the footpath.

Solution: Impulse is equal to change in momentum

impulse $\vec{l} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i = m[\vec{v}_f - \vec{v}_i]$ initial velocity is in opposite direction of final velocity, $m = 300g$, $v_r = 30m/s$, $v_i = 40m/s$

I = $0.3 \times [30 - (-40)] = 0.3 \times 70 = 21 \text{kg m/s}$, So, impulse on the footpath is 21kg m/s

2.3 **CIRCULAR MOTION**

Interesting Facts

When a body moves along a circle then force is required towards the centre of the circle to maintain circular motion. For example, the earth revolves around the sun then centripetal force is provided by gravitational force between earth and sun. The force acting toward the centre is called centripetal force and the equal and opposite force acting outward is known as centrifugal force.

2.3.1 Difinition of Angular Physical Quantities

When a particle moves along a circular path then its motion is called circular motion.

Angular position (a)

Consider a particle A is moving along a circular path of radius 'r'. Let the centre of the circular path lies at origin O and OX is along the X-axis. The initial position of particle A is given by angle AOX = θ , angle θ is known as the angular position of the particle.

Angular displacement (b)

As particle A moves from A to B in time interval Δt , angle increases by $\Delta \theta = (\theta + \Delta \theta - \theta)$. So, angular displacement is equal to angle AOB = angle BOX – angle AOX = $\Delta\theta$ through which the position vector of a moving particle traced or rotated in a given time interval it is called angular displacement of particle. Angular displacement is a vector quantity it is positive while it measured in the anti-clockwise direction and negative in the clockwise direction.

Angular velocity ω (c)

The rate of change of the angular position is known as angular velocity.

Thus, average angular velocity
$$
\omega_{avg.} = \frac{\text{total angular displacement}}{\text{total time taken}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}
$$

Instantaneous angular velocity $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$
 ω is a vector quantity, its direction is normal to plain of circular motion

and given by right-hand thumb rule. Unit of ω is rad s⁻¹.

(d) Angular acceleration (α)

The rate of change of angular velocity is known as angular acceleration.

Average angular acceleration
$$
\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta \omega}{\Delta t}
$$

Instantaneous angular acceleration $\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$

 α is a vector quantity, its direction is given by the right-hand thumb rule, which is normal to the plane of circular motion. Unit of α is rad s⁻².

The equations for constant angular acceleration α are given below

(1)
$$
\omega = \omega_0 + \alpha t
$$
 (2) $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$ (3) $\omega^2 = \omega_0^2 + 2\alpha t$

Here ω_0 is initial angular velocity at time t = 0, ω is final angular velocity at any time t, θ is angular displacement.

(e) Frequency (f) and time period (T)

Frequency (f) : The number of rotation or cycle per second in circular motion is called frequency. Unit of frequency is cycle per sec or hertz (Hz).

Time period (T): Total time taken by the particle in one complete rotation along a circular path is called time period. Frequency $f = 1/T$

A. Relation between Linear Velocity (v) and Angular Velocity (ω) $2.3.2$

Let the linear displacement covered by a particle along a circular path is AB in time interval Δt then

$$
\Delta s = r \Delta \theta \text{ divided both sides by } \Delta t, \left(\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t}\right) \text{ or } v = r\omega \tag{4}
$$

Here $v = \frac{\Delta s}{\Delta t}$ is the linear velocity of the particle, r is the radius of a circle and (Angle = $\frac{\text{arc}}{\text{radius}}$ or

 $\Delta\theta = \frac{\Delta s}{r}$, $\frac{\Delta\theta}{\Delta t} = \omega$ is the angular velocity of the particle

In vector form, this relation can be written $\vec{v} = \vec{\omega} \times \vec{R}$ here, \vec{R} is the position vector of a particle.

Relation between Linear Acceleration (a) and Angular Acceleration (α) В.

Linear acceleration of the particle is given by the rate of change of velocity. In a circular motion, it is also called tangential acceleration which acts along the tangent on the circle and given as

$$
a_t = \frac{dv}{dt} = \frac{d(r \omega)}{dt} = r \frac{d\omega}{dt} = r\alpha \ (v = r \omega) \text{ and } \frac{d\omega}{dt} = \alpha \text{ is the angular acceleration of the particle,}
$$

a_r is a component of total acceleration along tangent at any point of circular motion. Vector form of this relation can be written, $\vec{a}_t = \vec{\alpha} \times \vec{R}$ here, \vec{R} is the position vector of the particle.

Example: Determine the angular velocity of a particle that moves along a circular path of radius 40cm with a linear speed of 5 m/s.

Solution: Angular velocity $\omega = \frac{\text{linear velocity}}{\text{radius}} = \frac{v}{r} = \frac{5}{40 \times 10^{-2}} = 12.5 \text{ rad/s}$

Example: Find angular acceleration of a particle that is moving along a circular path of radius 40cm and speed of particle changes from 10m/s to 20m/s in 5.0 seconds.

Solution: The tangential acceleration of a particle is given by, $a_t = \frac{v_2 - v_1}{t_2 - t_1} = \frac{20 - 10}{5} = 2 \text{m/s}^2$

The angular acceleration $\alpha = \frac{a_t}{r} = \frac{2}{40 \times 10^{-2}} = \frac{200}{40} = 5$ rad/s²

Types of circular motion

 $\mathbf{1}$ Uniform circular motion: when the speed of a particle remains constant along the circular path then the motion of a particle is called uniform circular motion. In this case tangential acceleration of a particle $a_t = \frac{v_2 - v_1}{t_2 - t_1} = 0$ here $|\vec{v}_1| = |\vec{v}_2| = v = constant$

Thus, the acceleration of a particle is only toward the centre that is called centripetal acceleration and the magnitude of centripetal acceleration is given as follow

$$
a_r = \omega^2 r = \frac{v^2}{r^2} r = v^2 / r \ (\omega = \frac{v}{r})
$$

In uniform circular motion speed of the particle remains constant but direction changes at each point on the circular path due to that velocity changes and it is the cause of acceleration.

 $\overline{2}$ Non-uniform circular motion: when a particle moves along a circle with variable speed them both radial and tangential part of acceleration are considered.

Tangential acceleration
$$
a_t = \frac{v_2 - v_1}{t_2 - t_1} = \frac{dv}{dt}
$$

Radial or centripetal acceleration $a_r = -\omega^2 r = -\frac{v}{r^2}r = -v^2/r$

The magnitude of total acceleration $|\vec{a}| = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$

If total acceleration \vec{a} makes an angle θ with a_r then, $\tan \theta = a_l/a_r$

Centripetal and Centrifugal Force $2.3.3$

Α. **Centripetal force**

When a particle moves along a circle with constant speed then force acts towards the centre of the circle is provided with some external source of another kind of force (like tension, friction force etc). Acceleration of a particle toward the centre is given by v^2/r , so, force act on the particle towards centre is $F = ma = mv^2/r = m\omega^2 r$, (here $v = r\omega$) this force acting towards the centre is called centripetal force.

B. Centrifugal force

In non-inertial (rotating or accelerated) frames, newton's laws of motion are not obeyed. In this type of frames, pseudo force $(-m\vec{a})$ is experienced by the particle of mass 'm' and \vec{a} is the acceleration of the frame. If a reference frame rotating with constant angular velocity ω along a circular path of radius 'r'. Then in rotating, frame acceleration acts toward the centre of the circle with magnitude $\omega^2 r = v^2/r$. Now a particle of mass 'm' placed in this non-inertial reference frame then a pseudo force $(-\text{m}\omega^2 r)$ will have to be applied on this particle and the direction of pseudo force is away from the centre. That pseudo force is known as the centrifugal force which is equal and opposite to centripetal force.

Applications of Centripetal and Centrifugal Force 2.3.4

A. Banking of roads

On a circular part of the road, friction is not always reliable when high speed and sharp turns are involved. To avoid friction dependency, the roads are banked at a turn in which the outer edge of the road is raised at some height compared to the inner edge. This fact is known as banking of the road. By banking of road centripetal force is provided to vehicles to pass through the turn safely. Let bank angle (the angle at which the outer edge of the road is lifted compared to the inner edge) is θ and the weight of the vehicle is $W = mg$. Suppose the normal reaction of force (force perpendicular to the inclined plane) on the vehicle is N.

Then horizontal component of N is N sin θ provided the centripetal force to the vehicle while vertical component N cos θ is balanced the weight of the vehicle. Thus, N sin $\theta = mv^2/r$ and N cos $\theta = mg$

$$
\tan \theta = \frac{v^2}{rg} \text{ or } v = \sqrt{rg \tan \theta}
$$

v is speed of a vehicle (car or bus) with which it does not slide even on the smooth banked road.

 $\tan \theta = h/b$ ('h' is the height of the outer edge of the road compared to the inner edge and 'b' is the breadth of road)

Bending of cyclist В.

A cyclist has to be inclined by small-angle θ from his vertical position while crosses turn on a road. Let normal reaction on the tyre of a bicycle is N then the horizontal component of N is $N\sin\theta$ provided centripetal force and vertical component Ncos θ is balanced the weight of the cyclist.

$$
N\sin\theta = mv^2/r \text{ and } N\cos\theta = mg
$$

Thus, $\tan\theta = \frac{v^2}{rg}$

Applications (Real-life /Industrial)

- 1. When driving a car around a circular path then centripetal force is provided by banking in the road as well as friction between tyre and the road.
- 2. The dryer of a washing machine is experiencing centripetal force by the electric motor.
- 3. The centripetal force required in the revolution of an electron around the nucleus is provided by the electrostatic force between electron and protons lie in the nucleus.
- 4. When a moving bus suddenly takes a turn towards the left, a passenger in the bus is experienced an outward push to the right due to centrifugal force.
- 5. While mixing in a mixer or juicer, then centrifugal force is experienced outward.

Case-Study (Environmental/sustainability/social/ethical issue)

An artificial satellite revolves around the earth then required centripetal force is provided by gravitational force between the earth and satellite.

$$
\frac{mv^2}{r} = \frac{GMm}{r^2}
$$

The velocity of the satellite is given as $v = \sqrt{\frac{Gm}{r}}$ where, $G =$ gravitational constant, M is the mass

of earth and r is the radius of a circular orbit of a satellite. Angular velocity $\omega = \frac{v}{r}$

Create Inquisitiveness and Curiosity

When a stone tied to the end of the string is rotated by a person in a horizontal circle by provided centripetal force at another end of the string. Discuss the motion of the stone if the string is suddenly braked.

Solved Problem

Problem-1: A particle is moving in a circle of radius 40cm with uniform speed and it completes one rotation in 10s. Find the magnitude of the acceleration of the particle.

Solution: Angular velocity $\omega = \frac{2\pi}{T} = \frac{2\pi}{10}$ (time period = 10s, r = 40cm = 0.40m) centripetal acceleration $a_r = \omega^2 r = \left(\frac{2\pi}{10}\right)^2 \times 0.4 = 0.16$ m/s²

Problem-2: A block of mass 1.5kg is tied to a string of length 50cm, the other end of which is fixed. If the block is moved on a smooth horizontal table in a circle with a constant speed of 4m/s then find the tension in the string.

Solution: In this case, centripetal force is provided by the tension in the string

$$
T = \frac{mv^2}{r} = \frac{1.5 \times 4 \times 4}{0.5} = 48N
$$

Problem-3: A fan is rotating with a frequency of 20rev/sec. If the fan is switched off and it takes 2 minutes to stop (1) determine the value of angular retardation (2) find the total number of revolution made before it comes to rest.

Solution: (1) $\omega = \omega_0 + \alpha t$ and initial angular velocity $\omega_0 = 2\pi n = 2 \times 3.14 \times 20 = 125.6$ rad/s

$$
0 = 125.6 + \alpha(2 \times 60) \text{ thus, } \alpha = \frac{-125.6}{120} = -1.05 \text{rad/s}^2
$$

(2)
$$
\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = \left(\frac{\omega_0 + \omega}{2}\right) t = \left(\frac{125.6 + 0}{2}\right) \times 2 \times 60 = 125.6 \times 60 = 7536 \text{ rad}
$$

rev. = 7536/2 π = 7537/6.28 = 1200 revolutions

Problem-4: A car is moving on a banked road (or circular track) at an average speed of 90km/hr and the radius of the circular track is 250m. Determine the angle of banking (no friction on the road).

Solution:
$$
\tan \theta = \frac{v^2}{rg} = \frac{25 \times 25}{250 \times 10} = \frac{1}{4}
$$
 (average speed v = 90 × 1000/60 × 60 = 25m/s)

 $\theta = \tan^{-1}(0.25) = 14^{\circ}$

SUMMARY

- Scalar quantities are required only magnitude and vector quantities are represented by magnitude and direction both. Vectors are added by triangle law.
- Vector algebra consists of addition, subtraction, product and resolution of vectors.
- Force is responsible for the change in the state of rest and motion of a body.
- Momentum is given as the product of the mass and velocity of a body.
- \bullet Impulse is given as the product of force and time interval for which force acts. It is also measured by the change in linear momentum.
- A body moves along a circle then angular displacement, angular velocity and angular acceleration are defined. $v = r\omega$ and $a = r\alpha$ are the relations between linear and angular physical quantities.
- The centripetal force acts towards the centre and centrifugal force away from the centre of a circle both have the same magnitude mv^2/r .

EXERCISES

Α. **Objective Questions**

 2.1 Vector \hat{A} making angle 30° with horizontal and its magnitude is 6. Then rectangular vertical component of this vector is: $[LOD1]$

a. 3 b.
$$
3\sqrt{3}
$$
 c. 6 d. $\frac{1}{2}$

 $\vec{A} = 2\vec{i} + 5\vec{j} - 4\vec{k}$ and $\vec{B} = 3\vec{i} + p\vec{j} + 4\vec{k}$ are two vectors and vector \vec{A} is perpendicular to \vec{B} . Then $2.2.$ the value of p is: $[LOD2]$

a. 4 $b.2$ $c.10$

The angle between $\vec{A} = 2\vec{i} + 2\vec{j} - \vec{k}$ and $\vec{B} = 8\vec{i} - 6\vec{j}$ is: 2.3 $[LOD3]$

a.
$$
\cos^{-1}\frac{2}{15}
$$
 b. $\cos^{-1}\frac{5}{5}$ c. $\cos^{-1}\frac{1}{15}$ d. $\cos^{-1}\frac{1}{5}$

2.4 A body of mass m is kept on the inclined plane which makes an angle θ with horizontal then normal force on the body is: $[LOD2]$ a. mg sin θ

 $b. mg cos $\theta$$ d. mg tan θ c. mg

 $d.5$

a. $5\sqrt{2}$ b. 5 c. 10 d. $3\sqrt{2}$

(A) Answers of Objective Questions

B. Subjective Questions

$$
2.2 \quad \text{If } A = 2i + 3j \qquad B = 3i + 4j
$$

Then, find the magnitude of (a) $\vec{A} + \vec{B}$ (b) $\vec{A} - \vec{B}$, and (c) $\vec{A} \times \vec{B}$

 $[LOD1]$

 $[LOD1]$

- 2.3 A rocket ejected gases 1/100 of its initial mass with a relative speed of 2×10^3 m/s in one second just after launching it. Find the rocket's initial acceleration. $[LOD3]$
- 2.4 State and derive the law of conservation of linear momentum. $[LOD1]$
- 2.5° Find the expression of maximum speed on a banked road.
- If two vectors $\vec{A} = 2\vec{i} + \vec{j} \vec{k}$, $\vec{B} = \vec{i} 4\vec{k}$. Show that dot product of $\vec{A} \times \vec{B}$ with \vec{A} and \vec{B} is zero. 2.6 Write the conclusion of the result of the dot product. $[LOD3]$
- 2.7 A body rotated along a circular path of radius 10m. If the mass of the particle is 200g and the frequency of rotation is 10Hz. Find centripetal force required. $[LOD2]$
- 2.8 Short answer type
	- Define centripetal force. $1.$
	- $2.$ Define the recoil velocity of the gun.
	- $3₁$ Write the formula of impulse and its unit

(B) Answers of Subjective Questions

A2.2. a) $\sqrt{74}$ b) $\sqrt{2}$ $c)$ 1

A2.3. a = 20 m/s² (Hint a = $\frac{v_{rel}}{m} \times \frac{dm}{dt} = \frac{2 \times 10^3 m_0}{m_0.100} = 20$ m/s²] where m₀ is initial mass of the rocket $(t = 1s)$

A2.6. $\vec{A} \times \vec{B}$ is perpendicular to both \vec{A} and \vec{B} , (Because $\vec{A} \times \vec{B}$). $\vec{A} = (\vec{A} \times \vec{B})$. $\vec{B} = 0$ has proved) A2.7. $F = 7888 \text{ N}$ $F = m\omega^2 r$, $\omega = 2\pi f$

PRACTICAL

To verify triangle and parallelogram law of forces. 1.

Practical significance

Students can use triangle and parallelogram law for the addition of two vector quantities. They also verify the commutative law of vector addition.

Relevant theory

Triangle law: When two forces are represented both in magnitude and direction by two side of the triangle in the same order then the third side shows their vector sum in the opposite order both in magnitude and direction.

Parallelogram law: If two forces P and Q are represented both in magnitude and direction by two adjacent sides of a parallelogram, then their vector sum R is represented by both in magnitude and direction by diagonal which passes through the point of intersection of two forces.

The formula used: Lami¼s theorem is given as $\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R_1}{\sin \gamma}$ (for triangle law)

 $\vec{P} + \vec{O} + \vec{R} = 0$ if \vec{P}, \vec{Q} and \vec{R} force along the sides of a triangle in the same order.

The magnitude of resultant force $R_1 = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$ (for parallelogram law)

Practical outcomes (PrO)

PrO1: Apply triangle law and parallelogram law of vector addition

- PrO2: Verify triangles law of vector addition
- PrO3: Verify parallelogram law of vector addition

Practical setup (drawing/sketch/circuit diagram/work Situation)

Fig. 2.20: Gravesand's apparatus

Resources required

Precautions

- Hangers and weights should not touch the vertical board and the wall. 1.
- $2.$ There should be only one central knot on the thread which should be small.
- $3.$ While calculating the total force in each case the weight of the hanger should be added to the slotted weight into the hanger.
- $4.$ Make sure that all the hangers are at rest when the lines of action of forces are marked.
- All the pulleys should be free from friction or least friction so that friction can ignore. $5.$

Suggested procedure

- Board of parallelogram apparatus (or Gravesand's apparatus) shown in (Fig. 2.20) adjust 1. plumbline to keep vertical. Pulleys P_1 and Q_1 should be frictionless use oil if required.
- With help of drawing pins fix a white drawing paper sheet to the board. 2.
- $\overline{3}$. To make a junction point O of three pieces of threads tie one end of each of them in form of a knot.
- $4.$ The two weight hangers with the slotted weights using the other ends of two threads and pass over pulleys P_1 and Q_1 . Let P and Q be suspended weights (hanger + slotted weight).
- Now tie the weight (hanger + slotted weight) denoted by R with the end of the third thread 5. suspend vertically in the middle.
- Three weight P, Q and R acts as the three forces along the three threads at junction O in 6. equilibrium position.
- 7. Adjust the setup such that all the three weights hang freely and none of them touches the board.
- 8. Slightly disturb weights P and Q, leave them to settle in rest position. After settling the weights note the position of junction point O.
- 9. Now put the mirror strip lengthwise under each thread, make the position of the ends of the image of the thread in the mirror, covering the image by the thread. These new positions are P_1 and P_2 for the thread with the weight P and Q₁, Q₂ for the thread with the weight Q and R_1 , R_2 for the thread with the weight R.
- 10. Remove the paper from the board and draw lines with help of a half-meter scale through the points P₁ and P₂ to represent P, through points Q_1 and Q_2 to represent Q and through the points R_1 and R_2 to represent R. These lines are represented equilibrium forces. So must meet at point O.
- 11. Assuming a scale of 1cm=50g; OA=4cm and OC = 5cm to represent $P = 200g$ and $Q = 250g$.
- 12. Complete parallelogram OABC using set squares and meet OB. This OB, represent the vector sum of P and Q (forces) which corresponds to weight R.
- 13. Measure OB and multiply it by scale (1cm=50g) to find the resultant value of weight that is R_1 , it should be equal to already known weight R then parallelogram law verified.
- 14. To take different sets of observations change P and Q suitably.

TRIANGLE LAW OF FORCES

Graphical method

Use Bow's notation to name the forces P, Q, R as A'B', B'C', and C'A'. Select a suitable scale and draw the line A'B' parallel to force P and cut it equal to the magnitude of P from A' draw the line B'C' parallel to force Q and cut it equal to the magnitude of Q (Fig.2.22). Calculate the magnitude of C'A' i.e., R_1 which will be equal to the third force R which proves the triangle law of forces.

If R_1 differs from the original magnitude of R, the percentage error is found as follows:

Percentage error =
$$
\frac{(R - R_1)}{R} \times 100
$$

Analytical method

Measure angles α , β and γ and by using Lami's theorem check the following relation $rac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R_1}{\sin \gamma}$ and find R_1 , then percentage error = $\frac{(R - R_1)}{R} \times 100$

Parallelogram Law of Forces Graphical method

In (Fig. 2.23), cut $OA = P$ and $OC = Q$ in suitable scale. From A draw AB parallel to OC and BC parallel to OA. R_1 represents the resultant of force P and Q. As the system is in equilibrium it must be equal to R. Note that R and R_1 are in opposite directions.

Find percentage error =
$$
\frac{(R - R_1)}{R} \times 100
$$

Analytical method

Measure angle θ between force P and Q, then calculate R_1 using the formula of resultant given as $R_1 = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$ here θ is the angle between force P and Q.

Observations and calculations

Calculations

By Lami's theorem, resultant force R_1 is calculated as

$$
R_1 = \left(\frac{P}{\sin \alpha}\right) \times \sin \gamma \text{ or } R_1 = \left(\frac{Q}{\sin \beta}\right) \times \sin \gamma
$$

and using parallelogram law resultant force is calculated by the formula given as $R_1 = \sqrt{p^2 + Q^2 + 2PQ\cos\theta}$

Results and/or interpretation

With help of this experiment triangle law and parallelogram law, both are verified and we can add any vector quantities using these two laws. Percentage errors in both analytical and graphical methods are calculated by the formula given as

percentage error = $\frac{(R - R_1)}{R} \times 100$

This error is negligible hence triangle law and parallelogram law are proved.

Conclusions and/or validation

(To be filled by student)

Practical related questions

- 1. State triangle law and parallelogram law of vector addition.
- $2.$ Explain the mean of rectangular components of a vector.
- $3.$ Discuss main sources of error in the experiment using Gravesand's apparatus.
- $\overline{4}$. Explain the difference between a scalar and a vector quantity.

Suggested assessment scheme

(To be filled by teacher)

The given performance indicators should serve as a guideline for assessment regarding process and product-related marks.

* Marks and percentage weightage for product and process assessment will be decided by the teacher.

KNOW MORE

Newton's first law of motion: when the net force on a body is zero then the acceleration of that body remains zero or velocity of a body remains constant or zero. This law is sometimes called the law of inertia.

Inertial reference frame: Reference frame is a frame in which the motion of a body is described. If newton's first law of motion is valid in a frame (or acceleration $a = 0$ if $F = 0$) that is called an inertial frame of reference.

Non-inertial reference frame: If the law of inertia does not obey in a reference frame (or a \neq o even F = 0) that is called a non-inertial frame of reference.

Innovative Practical/Projects/Activities

- To determine the weight of a block using parallelogram law of vectors. 1.
- $\overline{2}$. Verify polygon law of forces.
- 3. Verify Lami's theorem.

REFERENCES & SUGGESTED READINGS

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Simulation on parallelogram

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Suggested Learning Resources for Practical

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- G. L. Squires, Practical Physics, Cambridge University Press, 2001. $2.$
- 3. https://ophysics.com/k2.html
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Work, Power and Energy

UNIT SPECIFICS

This unit focusses on the following aspects of basic physics:

- Concepts of work and examples of types of work
- Concepts of friction, laws of limiting fiction, coefficient of friction, reducing friction
- Work done in moving an object on horizontal and inclined plane for rough and plane surfaces and applications.
- Concepts of energy and its units
- Conservation of mechanical energy for freely falling bodies and examples of transformation of energy
- Power and its units, power and work relationship, calculation of power

RATIONALE

Work, friction, energy, and power are important parameters to solve many engineering problems. Here, you will understand the concepts of these quantities and you will also be able to differentiate between their types. You will be able to evaluate the amount of work required to be done against friction on inclined or horizontal surfaces and relate them for engineering problems. You will also understand the concepts of potential and kinetic energy and the conservation of mechanical energy using some examples. Finally, you will study about the work and power relationship and be able to solve numerical examples based on these concepts.

PRE-REQUISITES

Physics – High School level Physics Mathematics – Basic Algebra Other – Basic knowledge of Computer

UNIT OUTCOMES

- U3-O1: Identify physical quantities namely, work, energy, frictional force, power with its units and dimensions
- U3-O2: Solve problems based on work for engineering applications.
- U3-O3: Determine coefficient of friction for horizontal and inclined surfaces in related applications
- U3-O4: Describe various forms of energy and power with suitable examples. Also, find workpower relationship to solve engineering problems.

Mapping of unit ourcomes with the course outcomes:

3.1 WORK

Interesting Facts

When we solve puzzles or use our smartphones for e-learning while sitting at one place, except for internal chemical energy generated from bio-chemical processes in our body being converted to electrical signals to and fro your brain, no surplus work is done physically.

3.1.1 Work- Introduction and Definition

Work and energy are interchangeably used in our daily life. However, work is defined precisely and scientifically in Physics and so is energy. When the force is applied on a body or an object in the form of push or pull, the effect of force can be seen in its displacement in the direction of applied force. For example, displacement of a box placed on a smooth horizontal flat surface in the direction of push or pull. The relation between force and displacement gives us the physical quantity, known as the work (W).

Work (W) = Force **(F)** • Displacement **(r)**

where, \mathbf{F} (or \mathbf{F}) and \mathbf{r} (or \mathbf{F}) represent vector quantities for force and displacement, and the dot product (or scalar product) of these vector quantities gives us a scalar quantity i.e., work (W) .

Definition: The work done (W) by the force on an object or a body is defined as the scalar product of the force (F) acting on a body in the direction of displacement and this displacement **(r)** Or simply as a product of the component of the force acting on a body in the direction of the displacement and magnitude of this displacement.

 \therefore Work $(W) = F \cdot r = F \cos(\theta)$. r

where $F \cos \theta$ is the component of constant force in the direction of the displacement (r) . Note that bold letters in physical quantities indicates that they are vector quantities. Now, if there is a change either in force **(F)** or in displacement **(r)** with respect to time, then the work becomes a time dependent quantity and it is then written as $W = \int F dr$.

Here, the applied force is measured in newton and displacement is measured in meter. The SI unit of work is joule (J), named after a British scientist James Prescott Joule. One joule is the work required to exert force of 1 newton (N) through a displacement of 1 meter (m). (i.e., $1 J = 1 N-m$). Work or energy has other alternative units, such as erg (in CGS), calorie (CGS), electron volt, kilowatt-hour.

$3.1.2$ **Work – Examples**

Examples of Work done: (a) Zero Work (b) Negative Work (c) Positive Work

(a) Zero Work: When applied force (F_A) is perpendicular (\perp) to displacement (r), then θ° becomes 90° and work (W) done is zero OR when applied Force, $F_A = 0$ OR net displacement, $r = 0$, work (W) done is zero. For example, work done when (1) walking forward while lifting some object (2) pushing a concrete wall merely using hands (3) a moving car is stopped by applying brakes (4) the Earth revolves around the Sun.

Fig. 3.1: Examples of (a) zero work (b) negative work and (c) positive work done.

- (b) Negative Work: When applied force (F_A) is antiparallel to displacement (r), then θ° becomes 180° and work (W) done is negative. For example, work done by (1) friction force or retarding force acting on a rolling ball in opposite direction to its displacement (2) force due to gravity acting on a ball thrown vertically upwards (3) force applied by an oar on water during boating.
- (c) Positive Work: When applied force (F_A) is parallel (||) to displacement (r), then θ° becomes 0° and work (W) done is positive. For example, work done by (1) force due to gravity acting on a ball falling on the ground (2) force applied by a plough on soil during tilling the land (3) force applied by either crane or a helicopter for lifting heavy objects from the ground.

Applications (Real Life / Industrial)

There is not a single activity or application that doesn't include work, it is an integral part of human life. When we do some physical activities that involves non-zero force or displacements, it is called as work done (either positive or negative). When electrons jump from one electronic state to another, the work is done and energy released or absorbed is measured in electron volts (eV). We pay electricity bill for the work done by charges measured in kilowatt-hours or units, due to potential difference.

Case-Study (Environmental/Sustainability/Social/Ethical Issues)

When a basketball player throws the ball in a basket, he does positive work on a ball and when it falls down, again positive work is done by the force of gravity on a ball. In a field hockey, when the opponent player stops the ball moving in one direction with his hockey stick, negative work is done on the ball which causes retardation in its speed temporarily.

Create Inquisitiveness and Curiosity

Devyani is coming from a supermarket in her car and she stops the car in her parking. She then lifts a bag of groceries from her car and walk towards her home in a four-storey building. She climbs up the stairs up to 1st storey with a bag of groceries and reaches the elevator and then uses elevator to

go 4th storey. Analyze this scenario and find out where and how the positive, negative and zero work is involved.

Solved Problems

Problem-1: If a porter uses 45 N force to lift a suitcase and walk while doing work of 1250 J, then calculate the distance covered by a porter with a suitcase.

Solution:

Here, F = 45 N; W = 1250 J and angle θ between force and displacement of a porter = 0. Since the porter lifts the suitcase and walks, the force and the displacement vectors both, are in the same direction, i.e., cos 0° = 1, and the work done is positive. According the definition of work, $W = F \cos(\theta)$. $r = F$ $\cos(0^{\circ})$, $r = F$, r

Therefore, $r = \frac{W}{F} = \frac{1250}{45} = 27.78$ meters

3.2 **FRICTION**

Interesting Facts

Leonardo da Vinci first introduced the laws of sliding friction in 1493 but they remained unknown since he didn't document them. These laws were later rediscovered by Guillaume Amontons in 1699 and are known as Amontons' 3 laws of dry friction.

$3.2.1$ **Friction-Concepts and Types**

The force of friction or simply the friction is a retarding force that opposes the relative motion of one object over the other solid object or surface. Friction performs negative work and causes heat at the point of contact. The unit of force of friction is newton or kgms⁻² and its dimensions are [M¹L¹T⁻²]. The types of force of friction are: (i) static friction (ii) dynamic or kinetic friction. There is also, a fluid friction due to relative motion between two consecutive layers of fluid and can be explained by 'viscosity'.

(1) Static friction

Static friction does not exist on its own. When the object is in contact with the surface, and the force is applied on an object, static friction comes into play. It is the force (f_n) that opposes motion of an object due to the applied force (F) up to a certain limit and tries to keep the object at rest. Therefore, it is called as static friction force. For initial small values of the applied force (F), the static friction force (f_{\bullet}) increases as the applied force increases until it reaches the maximum value however, in opposite direction. Hence, to overcome, the friction between an object and the surface, the applied force (F) should be larger than the static friction. If there is no friction (in case of ideally smooth surface), the object would start moving when the applied force (F) acts (i.e., impeding motion). Examples of static friction are: (1) a person does not slip while walking, running or jumping on the ground due this friction (2) A heavy wooden cupboard doesn't move for small applied force.

(2) Kinetic or dynamic friction

Kinetic or dynamic friction comes into play when the relative motion of an object in contact with the surface takes place due to applied force, however it does not depend on the velocity of the moving object. Kinetic friction is categorized in sliding and rolling friction depending on how the object moves on the surface: (a) Sliding Friction: If the object slides over the surface, like a car skidding on the road after applying brakes or an ice-skater coming to stop in the ice-rink, the sliding friction comes into existence. (b) Rolling Friction: If the object rolls over the surface, like a roller skater coming to a stop in skating rink or a golf-ball rolling over the grass before it stops, the friction between the rolling object and the surface in contact is known as rolling friction.

Fig. 3.2: Static friction (f_s) acting on a box resting on a surface, sliding friction $(f_{silqimo})$ and rolling friction $(f_{Bollino})$ when a box moves on a surface.

$3.2.2$ **Laws of Limiting Friction and Coefficient of Friction**

Limiting friction is the maximum value of static friction when the object from rest is just about to move. At any instant, the value of the normal force and the limiting friction are directly proportional. Its mathematical expression is: $f_{\text{limiting}} = \mu_{\text{limiting}} N$ (i.e., Limiting Friction = $\mu_{\text{limiting}} \times$ Normal Force (or normal reaction force). Here, $\mu_{\text{limiting}} =$ coefficient of limiting friction. Limiting friction is always opposite to the direction of motion of the object. And it is directly proportional to the mass of an object and roughness of the surface.

The relation between the force of static friction (f_e) and the normal force (N) can be written as, $f_s \le \mu_s N$ where, μ_s is the coefficient of static friction. For maximum or limiting static friction, $f_s = \mu_s N$. The relation between kinetic friction and the normal force (N) is given by $f_s = \mu_k N$ where, μ_k is the coefficient of kinetic friction which is unitless quantity. Coefficient of kinetic friction is less than that of static friction i.e., $\mu_k < \mu_s$.

Fig. 3.3: Friction (I f I) vs. applied force (IF_AI) and the limiting friction.

$3.2.3$ **Reducing Friction and its Engineering Applications**

Friction is necessary in case of lighting match sticks, walking on the floor, writing on the paper, having a good grip on objects like book, brakes in the car etc. But it is not always desirable. There are some ways to reduce the friction to avoid unwanted heating effects like fire accidents due to friction, unwanted noise pollution, wear and tear in several items such as shoes, auto parts etc., or efficiency reduction in machines or slowing down of many processes. These methods of reducing friction include (1) making smoother surfaces by grinding, or chemical etching (2) using semi-solid-paste like lubricants (for e.g., for metals parts used in heavy machineries or in automobiles) (3) making streamlined or aerodynamic body (for e.g., of bullet trains or cars) (4) reduction in pressure or weight of the object (for e.g. to reduce wear and tear of vehicle tyres) (5) reducing the contact between surfaces of two objects (for e.g., using magnetic levitation effect in maglev trains) (6) using fluid friction (7) using rolling friction instead of sliding friction (for e.g., use of ball bearings).

3.2.4 Work Done Against Friction with Related Applications

(a) Force of friction on horizontal surface

For a block of mass 'm' lying on the horizontal surface of table or floor, we want to find work done against the force of friction. Consider that the force (F_A) is applied to the block which tries to move the block. But due to the force of static friction, block does not move. The force of static friction (f) can be written as $f_s = \mu_s N$ where, μ_s is the co-efficient of static friction. Here, N is the normal force exerted by the surface on the block and is equal to weight of the block according to Newton's 3rd law. Therefore, the normal force can be defined as $N = mg$. Hence,

$$
f_s = \mu_s \ N = \mu_s \ mg
$$

Here, we have considered only the magnitudes of the forces.

Now, suppose the external force (F_A) applied on the block is large enough to overcome the static friction and displaces the block. The block then starts moving with acceleration 'a' and experiences the force of kinetic friction (f_i). Then according to Newton's 3rd law of motion, the net force acting on the block with which it moves is

$$
ma = F_A - f_k
$$

And according to the laws of friction, we can write

$$
f_k = \mu_k \ N = \mu_k \ mg
$$

Here, μ_k is the coefficient of kinetic friction. So, the net force on the block is,

$$
ma = F_A - f_k = F_A - \mu_k mg
$$

Therefore, the acceleration of the block is, $a = \frac{F_A}{m} - \mu_k g$

And by the definition of work, we can write the work done by the force of friction as

$$
W = f_{\mathbf{k}} \cdot \mathbf{r} = f_{\mathbf{k}} r \qquad (\because \cos 0^{\circ} = 1 \text{ since } f_{\mathbf{k}} \parallel \mathbf{r})
$$

(b) Force of friction on inclined surface

Consider the case where the block is kept motionless on an inclined plane which makes an angle θ with the horizontal. Here, the weight of the block has two components – one parallel to the surface of the inclined plane and the other perpendicular to the surface of the inclined plane.

Perpendicular component of weight = W_{\perp} = mg cos θ = N (according to Newton's 3rd law) and from the laws of friction, $f_s = \mu_s N = \mu_s mg \cos \theta$. Parallel component of weight = W_{\parallel} = mg sin θ .

Fig. 3.4: A block on an inclined plane.

Now, suppose the block starts to move down towards wedge with acceleration 'a', then again from the laws of friction, we can write the expression for kinetic friction force (f_k) as

$$
f_k = \mu_k \ N = \mu_k \ mg \cos \theta
$$

From the Newton's $2nd$ law of motion, we can write

$$
ma = mg \sin \theta - f_k \text{ OR } ma = mg \sin \theta - \mu_k mg \cos \theta \text{ OR } \mu_k = \frac{Q \cos \theta}{\cos \theta}
$$

For zero acceleration (i.e., constant velocity), coefficient of kinetic friction = coefficient of static friction. Therefore,

$$
\mu_{k} = \mu_{s} = \frac{(g \sin \theta - 0)}{g \cos \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta
$$

This means that measuring the angle of inclination just before the object starts sliding gives the magnitude of coefficient of static friction. Applications of friction on horizontal and inclined plane are discussed in solved problems.

Applications (Real Life / Industrial)

 $(g \sin \theta - a)$

Our activities like brushing the teeth, walking, swimming in water, sliding, writing on a paper, fixing the nail to any surface, mopping the surface, generating static electricity by rubbing two woods or stones, scrolling the touch screen, leaning against the walk, applying brakes in our vehicles or fetching water from water wells using pulley etc. are all possible to perform because of friction.

Case-Study (Environmental/Sustainability/Social/Ethical Issues)

Friction stir welding is a solid-state joining process and it is used in train, aerospace application and modern shipbuilding. This method is used to join workpieces without melting them and heat is generated due to friction between superfast rotating tool and the workpiece. NASA has used this process in the most capable and powerful rocket, Space Launch System (SLS), whose 1st launch is schedule for 4th November, 2021.

Create Inquisitiveness and Curiosity

Which type of friction exists between your finger and touchscreen of your cellphone?

Solved Problems

Problem-1: A skier of 60 kg rests on an inclined surface of the mountain. The inclination angle θ is 30° with the horizontal, and the skier just begins to slide. What is the coefficient of static friction between the skier and the surface of the mountain? If the friction force is 50 N, find the coefficient of kinetic friction.

Solution:

Here, for $\theta = \theta_{max} = 30^\circ$. This is an angle for maximum static friction. For value of θ just above θ_{max} the skier begins to slide down. So, $\mu_s = \tan \theta_{\text{max}}$

 $\therefore \mu_{\rm s} = \tan \theta_{\rm max} = \tan (30^{\circ}) = 0.51$

For kinetic friction, $f_k = \mu_k N$. Therefore, $N = mg \cos \theta = \frac{I_k}{\mu_k}$

$$
\therefore \mu_{k} = \frac{f_{k}}{mg\cos\theta} = \frac{50}{60 \times 9.8 \times \cos(30^{\circ})} = 0.098.
$$
 Note here that $\mu_{k} < \mu_{s}$.

ENERGY AND POWER 3.3

Interesting Facts

An earthquake of magnitude 9.0 on Richter scale releases seismic energy of the order of 2.0×10^{18} joule $=$ 556 Terawatt $=$ 50 million TNTs. An increase in 1 magnitude results into approximately 30-fold increase in seismic energy released during earthquake.

Energy – Introduction, Kinetic Energy and Potential Energy $3.3.1$

Energy is the ability to carry out work. Energy is a quantitative physical property that is transferred to some object for carrying out work. Energy is a scalar quantity and its SI unit is joule or N-m which is same as that of work. Other measurements units are erg (or dyne/cm), calorie, kilowatt-hour (kWh), electron volt (eV) or British thermal unit (BTU) .

Energy in its any form is either kinetic energy (KE) or potential energy (PE).

Kinetic energy (KE): When an object with mass 'm' is in motion with some velocity 'v', it is said

to possess kinetic energy (KE) = $\frac{1}{2}$ mv². Note here, that kinetic energy varies with mass (m) and square of velocity (v) . It is a scalar quantity and it measures the ability of a moving object to perform work. Fast moving ball thrown by a fast pace baller, bullet fired from the gun, a car running at speed of 120 kmph, landslide in mountain region or glacier avalanche are some of the examples of objects having high kinetic energy either due to high velocity or large mass.

Potential energy (PE): It is an ability to perform work when converted into kinetic energy. It is a stored energy in an object or system of several objects. For example, compressed spring in clocks stores its energy in the form of potential energy. PE is a scalar quantity.

Here, we shall consider only gravitational potential energy.

Gravitational Potential Energy $3.3.2$

When an object of mass (m) is brought from infinity to point inside the gravitational field of some source mass (M) with constant speed, the amount of work done in displacing the object into the gravitational field of source mass is stored in the form of potential energy and is known as gravitational potential energy. It is denoted by 'U' or PE.

Gravitational potential energy, $U = m g \Delta h$, where m = mass of the object (kg), g = gravitational acceleration = 9.8 m/s², and Δh = height of an object above the ground (m). Examples of gravitational potential energy are: (1) Water stored in dam at height 'h' (2) Swimmer on diving board before he jumps into swimming pool (3) Coconut or any fruit on the tree before it falls (4) thick layer of ice at the peak of the Himalayas (5) a cleaning worker on the Burj Khalifa etc.

Derivation of gravitational potential energy

Let the source mass be 'M' and let it be placed at a point along the X-axis. Initially the test mass 'm' is placed at infinity (∞) . Under the influence of constant gravitational field or force (F) of source mass (M), test mass (m) displaces by a very small amount (dx) along X-axis. i.e., a small amount of work (dW) is done on the test mass (m) . This can be written as

 $dW = F dx$.

Note here, that the gravitational force (F) is attractive and the displacement of the test mass (m) is towards the source mass (M) i.e., in negative X-axis direction. This means that F and dx are parallel to each other.

Therefore, $dW = -G \frac{mM}{r^2} dx$.

If we integrate this equation on both the sides, we get

$$
W \quad = \quad -\int\limits_{-\infty}^{r} G\frac{mM}{x^2} dx = \left[G\frac{mM}{x} \right]_{-\infty}^{r} = \left[G\frac{mM}{r} \right] - \left[G\frac{mM}{\infty} \right] = \left[G\frac{mM}{r} \right]
$$

Since, the work done on the test mass (m) is stored as its potential energy (U) , we can rewrite above equation as

$$
U = - \left[G \frac{mM}{r} \right]
$$

This is the gravitational potential energy of a test mass (m) stored at a point with distance (r) from the source mass (M) .

Now, if the test mass (m) moves inside the gravitational field of a source mass (M) from one point (A) to another point (B). Let r_i be the position of test mass (m) for point (A) from the source mass (M) and r_f be the position of test mass (m) for point (B) from the source mass (M).

Then, the change in the gravitational potential energy for this test mass (m) is given by

$$
\Delta U = - \left[G \frac{mM}{r} \right]_{r_i}^{r_f} = - \left[G \frac{mM}{r_f} \right] + \left[G \frac{mM}{r_i} \right] = GmM \left[\frac{1}{r_i} - \frac{1}{r_f} \right]
$$

If $r_i = R$ and $r_f = R + h$, where h = small displacement of test mass (m) in the same gravitational field then

$$
\Delta U = \text{GmM} \left[\frac{1}{R} - \frac{1}{R+h} \right] = \text{GmM} \frac{h}{R(R+h)}
$$

Since $h \ll R$, $(R + h) \sim R$,

$$
\Delta U = \text{GmM}\frac{h}{R^2} = mgh \qquad \left(\because g = \frac{\text{GM}}{R^2}\right)
$$

Here, g = acceleration due to gravity. For $r_i > r_f$, ΔU is negative.

$3.3.3$ **Mechanical Energy and Conservation of Mechanical Energy**

Total mechanical energy of an object with its kinetic energy and potential energy is written as the sum

of kinetic energy and potential energy i.e., $\left(\frac{1}{2}mv^2 + mgh\right)$. It is the energy that describes the motion or position or both of the object.

Conservation of mechanical energy for a freely falling body

The total mechanical energy (TME) of a system is conserved i.e., the energy can neither be created nor be destroyed; it can only be internally converted from one form to another provided the forces acting to perform work on the system are conservative forces.

Let us understand this conservation law using the following example of a freely falling body under the influence of gravitational force of the Earth. \hat{g} is the acceleration due to gravity.

Fig. 3.5: Schematic diagram of a free fall of a ball

Let us consider that a ball of mass 'm' is dropped from a height H, as shown in figure. So,

(a) At height H: Potential energy (PE) = mgH and Kinetic energy (KE) = 0 (\cdot · V₀ = 0) Therefore, Total mechanical energy = $PE + KE = mgH + 0 = mgH$

(b) At height h: Potential energy (PE) = mgh and Kinetic energy (KE) = $\frac{1}{2}$ mV_t². Using the equations of motion, the velocity V_t at a height 'h' for an object of mass 'm' falling from a height 'H' can be written as $V_t = \sqrt{2g(H-h)}$. Therefore, the kinetic energy can be given as,

$$
\frac{1}{2}mV_t^2=\frac{1}{2}m\Big(\sqrt{2g(H-h)}\Big)^2=mgH-mgh
$$

- \therefore Total mechanical energy = PE + KE = mgh + (mgH mgh) = mgH
- (c) At height zero: Potential energy (PE) = 0 (\therefore H = 0) and Kinetic energy (KE) = $\frac{1}{2}$ mV².

Again, using the equations of motion, we get velocity of a freely falling body, $V = \sqrt{2gH}$ at the bottom just before touching the ground. Hence, the kinetic energy can be written as,

$$
KE = \frac{1}{2} mV^2 = \frac{1}{2} m \left(\sqrt{2gH}\right)^2 = \frac{1}{2} m(2gH) = mgH
$$

Therefore, Total mechanical energy = $PE + KE = 0 + mgH =$ mgH

Thus, we saw that the total mechanical energy (TME) of the system remains constant for freely falling body under effect of the gravitational force.

$3.3.4$ **Transformation of Energy**

According to conservation law of energy, energy can only be transformed from its one form to another and can't be destroyed or created. Following are few examples of transformation of energy:

- Potential Energy to Kinetic Energy: e.g., Waterfall, skydiver (1)
- (2) Kinetic Energy to Gravitational Potential Energy: e.g., Satellites at an instant of time
- (3) Gravitational Potential Energy to Electric Energy: e.g., Hydroelectric Dams
- (4) Kinetic Energy / Mechanical to Thermal/Heat Energy: e.g., Rubbing both the hands
- (5) Heat Energy to Mechanical Energy: e.g., Steam engine
- (6) Heat Energy to Electrical Energy: e.g., Thermocouple, Thermal power plants
- (7) Electric Energy to Heat Energy: e.g., Room heater, Water heater, toaster, oven
- (8) Mechanical Energy to Electric Energy: e.g., Electric Generator
- (9) Electric Energy to Mechanical Energy: e.g., Electric Motor, blender, juicer-mixer, fan
- (10) Electric Energy to Chemical Energy: e.g., Electroplating, charging Li-ion battery
- (11) Chemical Energy to Electric Energy: e.g., battery-powered torchlight, fuel cells
- (12) Chemical Energy to Electric and Mechanical Energy: e.g., Movement of human body
- (13) Chemical Energy to Heat Energy and Radiant Energy: e.g., Burning of wood or coal
- (14) Sound Energy to Electric Energy: e.g., Microphone
- (15) Electric Energy to Sound Energy: e.g., Loud speakers, sound amplifiers
- (16) Electric Energy to Heat Energy and Radiant Energy: e.g., An electric bulb
- (17) Elastic strain Energy to Electric Energy: e.g., In piezoelectric (gas lighters)
- (18) Wind Energy to Mechanical or Electric Energy: e.g., Windmills
- (19) Solar Energy (i.e., Electromagnetic Energy) to Chemical Energy: e.g., Photosynthesis
- (20) Solar Energy to Electric Energy: e.g., Solar cells or Photovoltaic cells

$3.3.5$ **Power and Its Units**

In Physics, power is the time rate at which work is done or energy is transferred. It shows that not only work is done but also how fast it is done. For example, climbing a mountain shows your physical fitness but how fast you climb indicates the power of your muscles, beating of your heart per minute shows that blood is being pumped into body but how fast or slow does it beat gives the idea about muscular pressure it has to withstand due to power output. Power is a scalar quantity. Its dimensions are $[ML^2T^{-3}]$. Its SI unit is watt (W) named after James Watt, who was one of the innovators of steam engine. 1 watt = 1 $J/s = 1$ Js⁻¹. Another unit of power which is still used in automobile industry is the horsepower (hp). 1hp = 746 W. The most recent all electric 'Tesla S' (2021) car offers power output of 1020 hp which is equal to 761kW. A typical automobile vehicle requires about 20 hp or 15 kW of power to keep it running at 80kmph. Electrical good and appliances like, bulb, refrigerators, blender, water heater etc. are measured in watt.

$3.3.6$ **Power and Work Relationship**

We can find average power (P_{avg}) by taking a ratio of the work to the total time 't' taken for work to be done i.e., $P_{avg} = \frac{W}{t}$. The instantaneous power is defined as

$$
P = \frac{dW}{dt} = \frac{F.dr}{dt} = F.v \qquad \qquad \left(\because W = F \cdot r \text{ and } \frac{dr}{dt} = v\right)
$$

Applications (Real Life / Industrial)

High energy water-jet cutting is used to cut soft materials like plastic foams, fibre glass or printed circuit boards, diapers, leather etc in industries. Moreover, energy in its various forms is fundamentally required wherever work is performed. All electrical appliances and electronic devices are rated in terms of power consumption. Automobile engines are power-rated which shows how fast the vehicle can pick up in few seconds. Majority of sports related activities such as kick boxing, weightlifting, tennis, cricket etc. are about power, precision and timing.

Case-Study (Environmental/Sustainability/Social/Ethical Issues)

Cryptocurrencies like bitcoin require computers that handle intense processing power for bitcoin mining. These special computers consume energy about 22 TWh to 110 TWh per year, which is as good as the yearly energy consumption of countries like Malaysia, Sweden or Ireland.

Create Inquisitiveness and Curiosity

How much power does an adult heart need, if an average heart beat is 75 per minute?

Solved Problems

Problem-1: Convert 24.5 GeV to joules. 1 eV = 1.6×10^{-19} J

Solution:

 $1 \text{ GeV} = 1.6 \times 10^{-19} \times 10^9 \text{ J} = 1.6 \times 10^{-10} \text{ J}$

Therefore, 24.5 GeV = 24.5 \times 1.6 \times 10⁻¹⁰ J = 39.2 \times 10⁻¹⁰ J

Problem-2: 55 kg and 52 kg athletes run the track of 200 meters in 19.19 sec and 21.34 sec respectively. Who has more muscular power? Consider $g = 9.8$ m/s².

Solution:

Here, $m_1 = 55kg$, $m_2 = 52 kg$, $g = 9.8 m/s^2$, $r = 200$ meters, $t_1 = 19.19$ sec and $t_2 = 21.34$ sec. Since, Power (P) = $\frac{W}{t}$ = $\frac{F \cdot r}{t}$ = $\frac{mg \cdot r}{v}$, we can calculate the muscular power for both athletes. $P_1 = \frac{mg \cdot r}{t} = \frac{55 \times 9.8 \times 200}{19.19} = 5617.51 \text{ W} = 5.618 \text{ kW}$ $P_2 = {mg.r \over t} = {52 \times 9.8 \times 200 \over 21.34}$ = 4776.00 W = 4.776 kW

The 55 kg athlete used more muscular power.

Problem-3: If a hydropower plant, nuclear power plant and thermal power plant generate maximum output of 4.3 GW, 850 MW and 2100 MW at full load for 6 hrs/day, 12hrs/day and 8 hrs/day respectively, then calculate the energy produced in an hour for each plant.

Solution:

Here, Power (P) = $\frac{W}{t}$. Therefore, W = P.t

(a) For hydropower plant: $P = 4.3$ GW = 4300 MW for 6 hrs/day

So, for 1 hr power output is, P = $\frac{4300}{6}$ MW = 716.67 MW

 \therefore Energy produced per hour = W = Pt = 716.67 \times 1 = 716.67 MWh

(b) For nuclear plant: $P = 850$ MW for 12 hrs/day

So, for 1 hr power output is, $P = \frac{850}{12} MW = 70.83 MW$

 \therefore Energy produced per hour = W = Pt = 70.83 \times 1 = 70.83 MWh

(c) For thermal power plant: $P = 2100$ MW for 8 hrs/day

So, for 1 hr power output is, $P = \frac{2100}{8} MW = 262.5 MW$

 \therefore Energy produced per hour = W = Pt = 262.5 \times 1 = 262.5 MWh

In a given example, hydropower plant provides more energy in an hour.

UNIT SUMMARY

- Both, work and energy have similar dimensions i.e., $[M¹L²T⁻²]$. \bullet
- Both kinetic energy and potential energy curves with respect to distance are parabolic.
- Friction depends on the roughness of the contact surface rather than its hardness. \bullet
- For two relatively stationary systems of objects in contact there is a static friction (f_c), while \bullet for two moving systems in contact, there exists kinetic friction (f_i) due to their relative motion.
- Coefficient of friction cannot exceed the value 1.0 \bullet
- Coefficient of kinetic friction is less than or equal to coefficient of static friction.
- For an object placed on inclined plane, static friction can be defined by measuring the angle of inclination.
- Work is the force applied multiplied by the displacement in the direction of force.
- Power is the work done in a unit time.
- Energy measures the ability to do work and power measures how fast the work is done.
- Other units for work and energy are : (a) 1 erg = 1 dyne/cm = 10^{-7} joule (J) (b) 1 calorie (cal) \bullet = 4.186 J (c) 1 electron volt (eV) = 1.6×10^{-19} J (d) 1 kilowatt-hour (kWh) = 3.6×10^{6} J (commercial unit).

EXERCISES

(A) **Objective Questions**

- A3.2. A (True)
- A3.3. D (fluid friction)

A3.4. D (rolling a stone on a polished marble surface)

A3.5. B (False)

A3.6. C (kinetic energy)

Subjective Questions (B)

- 3.1 List out the examples of positive, negative and zero work.
- 3.2 Compare static friction and kinetic friction.
- 3.3 If a volleyball player applies force of 24 N on a volleyball making an angle of (i) 30° (ii) 45° and (iii) 60° respectively with horizontal plane, and work done on a ball is 2500 J in all cases, find the displacement of a ball in each case. [LOD2]
- 3.4 Annie uses 2 Units of power supply for domestic purpose per day and the electricity board charges 1.25 Rs. per kWh, calculate the total energy spent per month and corresponding electricity bill amount. $[LOD2]$
- A gold bar with a mass of 10 kg rests on a plane inclined at 45° from horizontal. Find out the 3.5 component of the weight of a gold bar which is parallel to the inclined plane. $[LOD2]$
- 3.6 Analyse and discuss your results related to work done for this scenario: The needles in wrist watch move in an hour, raindrops falling on ground, walking on a tight rope at some height from the ground. [LOD3]

Answers of Subjective Questions (B)

- A3.3. 120.28 m, 147.31 m, 208.33 m [Hint: Work done, $W = F \cos\theta$. r]
- A3.4. Total energy spent in a month = 60 Units and electricity bill is 75 Rs. a month [∴ 2 Units/day \Rightarrow average 30 days \times 2 Units = 60 Units in a month = 60 kWh per month. Now, 1.25 Rs. are charged per Unit consumption. Therefore, for 60 Units, electricity bill will be 75 Rs.]
- A3.5. 69.3 N [Hint: $W_{||} = m g sin\theta$]

PRACTICALS

1. To find the coefficient of friction between wood and glass using a horizontal board.

Practical significance

The coefficient of limiting friction between two surfaces is the maximum value of static friction when the object from rest is just about to move. If its value is large, the force required to move the object is high. This coefficient of friction between any two unpolished horizontal surfaces can be determined using this simple experimental set up.

Relevant theory

For theory, refer to Unit-3 (Section: 3.2)

Formula: $f_{\text{limiting}} = \mu_{\text{limiting}} N$; where, $f_{\text{limiting}} =$ Limiting force of friction; $\mu_{\text{limiting}} =$ coefficient of limiting friction, and $N =$ Normal force

[LOD1] $[LOD1]$

Practical outcomes (PrO)

The practical outcomes are derived from the curriculum of this course:

- PrO1: Acquire skills in setting up the experiment and precise measurement of an instant when the wooden block just starts moving with or without masses.
- PrO2: Describe the force of limiting friction and determine the coefficient of limiting friction between glass and wood using a horizontal board.
- PrO3: Operate the required equipment with proper precautions either in a group or individually.

Practical setup (drawing/sketch/circuit diagram/work situation)

Fig: 3.6: Experimental setup for determining the coefficient of limiting friction

Resources required

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Precautions

- The horizontal surface of the board must be kept dust free. $1.$
- $2.$ Keep the connecting thread or a string horizontal. Thread must be thin, light, strong, unstretched and unspun.
- $3.$ Maintain the friction of pulley minimum by oiling it properly.
- $4.$ Tap the glass top (or table top) gently.
- 5. Always put the mass at the centre of the wooden block.
- 6. The surface of horizontal plane (i.e., glass top) and moving body must be kept dry.
- 7. Size of the block and masses should be chosen judiciously.

Suggested procedure

- $\mathbf{1}$. Using spring balance, measure the mass (M) of the given wooden block with hooks on its sides and the scale pan (m).
- $2.$ Place the glass on a horizontal board. Use spirit level to ensure that the board surface is horizontal. The board surface must be clean and dry.
- $3.$ Now, fix a frictionless pulley on one edge of board-top as shown in figure.
- $4.$ The one end of a string of to a scale pan and the other end to the hook of the wooden block.
- $5.$ Then place the wooden block on the horizontal glass plane and pass the string over the pulley such that it remains horizontal between wooden block and pulley.
- 6. Put some mass (q) on the scale pan. Tap the glass-top gently with your finger and check if the wooden block starts moving.
- 7. Increase the mass (q) on the scale pan gradually until the wooden block just starts moving on gently tapping the glass top. Note the total mass kept on the scale pan.
- 8. Place some known mass (p) in the centre of the wooden block and adjust the mass (q) on the scale pan so that the wooden block along with mass p just begins to slide on gently tapping the glass top. Note the values of p and q.
- Repeat step 9 for five more values of p and tabulate the corresponding values of q. 9.
- 10. Plot a graph of force of limiting friction $(f_{limiting})$ on y-axis vs. Normal force 'N' on x-axis and calculate the coefficient of limiting friction.

Observations and calculations

Observations:

- Mass of the scale pan, $(m) = \underline{\hspace{2cm}} g$ 1.
- $2.$ Mass of the wooden block $(M) = \underline{\hspace{2cm}} g$
- Acceleration due to gravity (g) at the place of experiment = μ/s^2 $3.$

Observation table:

Calculation

From theory: (calculate coefficient of limiting friction between glass and wood surfaces for various combinations of p and q masses and then take their average.)

 $f_{\text{limiting}} = \mu_{\text{limiting}} N$ $\therefore \mu_{\text{limiting}} \frac{f_{\text{limiting}}}{N} =$ From graph of force of limiting friction (f_{limiting}) on y-axis vs. normal force (N) on x-axis:

 $\mu_{\text{limiting}} = \frac{\Delta f_{\text{limiting}}}{\Delta N}$

Results and/or interpretation

The value of coefficient of limiting friction, μ_{limiting} between surface of wooden block and the glass top is:

- From theory (or calculation) = $\frac{1}{\sqrt{2}}$ 1.
- $\overline{2}$.

Conclusions and/or validation (To be filled by student)

Practical related questions (Use separate sheet for answer)

- $1.$ Why does the friction between two surfaces can never be zero?
- 2. What is the purpose of using unpolished surface in this experiment?
- $3.$ List out the applications where the force of friction is used.
- 4. What is the difference between static friction and kinetic friction?
- 5. What happens if we choose spherical body to study the limiting friction between the two surfaces?

Disposal of waste

Classify the waste materials to be thrown in this experiment in the following bins:

Environment friendly approach: reuse, reduce and recycle

Apparatus for this practical are reusables for sufficiently long time if they are operated with care.

Suggested assessment scheme (To be filled by teacher)

The given performance indicators should serve as a guideline for assessment regarding process and product related marks.

* Marks and percentage weightages for product and process assessment will be decided by the teacher.

To verify law of conservation of mechanical energy (PE to KE). $2.$

Practical significance

Total mechanical energy of a system is always conserved which means that energy can neither be created nor destroyed. It can be internally converted from one form to another form. This simple experiment verifies this law of conservation of mechanical energy. The potential energy of the steel ball is converted into its kinetic energy.

Relevant theory

For theory, refer to Unit-3 (Section: 3.3.1, 3.3.2, and 3.3.3)

Practical outcomes (PrO)

The practical outcomes are derived from the curriculum of this course:

- PrO1: Acquire skills in setting up the experiment and precise measurement of position mark of the rolling steel balls.
- PrO2: Describe and verify the law of conservation of total mechanical energy.
- PrO3: Operate the required equipment with proper precautions either in a group or individually.

Practical setup (drawing/sketch/circuit diagram/work situation)

Fig. 3.7: Experimental setup for verifying the law of conservation of mechanical energy using double inclined plane and a spherical steel ball.

Resources required

Precautions

- The steel ball and the inclined tracks must be completely clean. Use cotton moistened in benzene 1_{-} to clean the tracks.
- $2.$ Affix the planes at an inclination angle such that they remain fixed when the steel ball rolls onto them. Both tracks should be in same vertical plane.

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- The width between two inclined planes should be negligibly small as compared to distance $3.$ moved by the steel ball along the track.
- $4.$ Release the steel ball gently onto the track.
- $5₁$ Note the extreme positions of the ball on inclined planes immediately and accurately.

Suggested procedure

- Find a sufficiently large horizontal surface of a laboratory table. Measure the level of surface $1.$ using spirit level, if the need be.
- Arrange the double inclined track on the table as shown in the figure and ensure that they are $2.$ stable. Use weights on its wings to keep them stable during experiment.
- $3.$ Place the wooden blocks under each track such that both the tracks are inclined at some angle.
- Insert the wooden block under each track to make it inclined. Both the tracks may not be $\overline{4}$ inclined at the same angle.
- $5.$ Now, take the steel ball and put it on mark 'R' on inclined track 1 and leave it gently.
- When the steel ball reaches the highest point 'S' on inclined track 2, note that position. 6.
- Measure the vertical heights 'PR' and 'QS' using a plumb line and a metre scale. $7.$
- 8. Change the positions of 'R' by changing inclination angles of track 1 and repeat 5, 6, 7 few more times.
- Tabulate the observations. 9.

Observations and calculations

Observations

- $1.$ Least count of the stopwatch $=$ _____ sec
- $2.$ Diameter of a steel ball, $D = \underline{\hspace{2cm}}$ cm

Observation table

Calculation

Difference $(h_1 - h_2) =$ for each observation.

Results and/or interpretation

Conclusions and/or validation (To be filled by student)

Practical related questions (Use separate sheet for answer)

- $1.$ What is the interpretation of your results for this experiment?
- $2.$ Suggest an alternative method to verify law of conservation of energy.
- $\overline{3}$. List out the examples of conservation of energy in your field of study.
- $\overline{4}$. Explain the statement: Energy is neither created nor destroyed.
- 5. Give the different units for measuring energy.

Disposal of waste

Classify the waste materials to be thrown in this experiment in the following bins:

Environment friendly approach: reuse, reduce and recycle

Apparatus for this practical is reusable for sufficiently long time if it is operated with care.

Suggested assessment scheme (To be filled by teacher)

The given performance indicators should serve as a guideline for assessment regarding process and product related marks.

* Marks and percentage weightages for product and process assessment will be decided by the teacher.

KNOW MORE

Work-Energy Theorem: It is the theorem that shows the relation between work done and energy spent. This theorem states that the net work done on an object is equal to the change in kinetic energy of the object. This can be mathematically represented as, $KE_{final} - KE_{init} = W$, where $KE_{final} = Final$ kinetic energy, KE_{mit} = Initial kinetic energy and W = net work done. Work- Energy theorem indicates the conservation of energy according to which we can only transfer energy from one form to another.

Innovative Practical/Projects/Activities

- (1) Make a list of your 10 physical activities and figure out in which activity work done is positive, negative or zero.
- Find out the apparatus used to measure friction and its working. Present your findings in the (2) form of report or presentation.
- (3) You will need protector, 5 rupees coin and your book. Put the flat part of the coin on the book and tilt the book at an angle at which the coin just starts moving. Measure the angle of inclination and find the coefficient of kinetic friction.

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- 10. Swayam Prabha, INFLIBNET, India, https://www.swayamprabha.gov.in/index.php/
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Suggested Learning Resources for Practical

- 1. C. L. Arora, B.Sc.Practical Physics, S. Chand Limited, 2001.
- $\overline{2}$. G. L. Squires, Practical Physics, Cambridge University Press, 2001. https://www.learncbse.in/to-study-the-conservation-of-energy-of-a-ball-rolling-down-on-aninclined-plane/

Rotational Motion

UNIT SPECIFICS

This unit focusses on the following aspects of basic physics:

- Translational and rotational motions with examples
- Concepts of torque and angular momentum with examples
- Conservation of angular momentum and its applications
- Moment of inertia and its physical significance and the radius of gyration for a rigid body
- Theorems of parallel and perpendicular axes
- Moment of inertia of rod, disc, ring and sphere (hollow and solid)

RATIONALE

Motion is classified into three categories translational, rotational and vibrational motion. Description of translational and rotational motion is given with examples, torque and angular momentum and their relationship are explained. The concept of conservation of angular momentum and its uses are described. The moment of inertia and its importance is discussed. Statement of theorems of the moment of inertia and formulae of M.I of some objects are also given in this text.

PRE-REQUISITES

Physics – Basics of force and motion Mathematics – Basics of vector algebra and linear algebra Others – Basic knowledge of computer

UNIT OUTCOMES

U4-O1: Identify translational and rotational motion. Relate moment of inertia with torque and angular momentum. Apply conservation of angular momentum, use M.I. of different bodies and able to state theorems of the moment of inertia.

Mapping of unit outcomes with course outcomes:

4.1 **ROTATIONAL MOTION**

Interesting Facts

A skater extends or pulls in a leg its rotational inertia is changed but in translational motion, no such effect exists. In spinning top or gyroscope change in angular momentum by applied torque is not along the direction of angular momentum vector but it will be in a different direction. In rotational motion, torque plays the same role as a force in translational motion.

$4.1.1$ **Translational and Rotational Motions with Examples**

Δ **Translational motion**

The motion of a body is pure translational if the velocity of all particles of the body is the same at any instant of motion. During such motion, all particles have the same displacement (\vec{s}) , velocity (\vec{v}) and acceleration (\vec{a}) at an instant. Examples: the motion of a car along a straight line, the motion of a train on its track, a person walking on the road, a flying bird in the sky.

There are three types of translational motion

- (1) Linear or one-dimensional motion (1D): when a body is moving along a straight line then its motion is called linear motion. Examples: freely falling body, running bus on a straight road.
- (2) Two-dimensional motion (2D): when a body is moving in the plane then two coordinate changes either (x, y) or (y, z) or (z, x) . Examples: motion of a car on the zig-zag road, projectile motion.
- (3) Three-dimensional motion (3D): when a body is moving in the sky then all the three coordinates change that motion is called 3-D motion. Examples: flying birds, kite, airplane in the sky.

Let a system have n-particles of mass m_1 , m_2 , ------------------m_n. The body is moving in pure translational motion. Thus, $\vec{a}_1 = \vec{a}_2 = \vec{a}_3 \dots \vec{a}_n = \vec{a}$

and $\vec{v}_1 = \vec{v}_2 = \vec{v}_3$ $\vec{v}_n = \vec{v}$

By newton's law of motion $\vec{F}_{net} = m_1 \vec{a}_1 + m_2 \vec{a}_2 + = M \vec{a}$ (M is the total mass of the body) linear momentum.

$$
p = m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + \dots = M \vec{v}
$$

The total energy of the body-

$$
KE. = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \dots = \frac{1}{2}Mv^2
$$

B. Pure rotational motion

Let a rigid body rotated about a fixed-line (or axis) that line is called the axis of rotation and shown in fig (4.1) . Each point of the body moves in a circle whose centre lies on the axis of rotation and each point traces the same angle in a particular time interval. Such a motion is known as pure rotational motion. In pure rotation, each particle is rotated with the same angular velocity (since the body is rigid).

Thus,
$$
v_1 = \omega r_1
$$
, $v_2 = \omega r_2$, $v_n = \omega r_n$,

Fig. 4.1: Rotatinoal Motion
Examples of rotational motion

- In a ceiling fan, each point of its blades and body covering 1. motor rotates in circles and the centre of all these circles lies on a vertical line through the centre of the body is called the axis of rotation.
- $2.$ When we open or close the door each point of the door traces a circle and the centre of these circles lie on a vertical line that passes through the hinges of the door. That line is the axis of rotation.
- $3.$ Rotation of the earth about its axis.
- The motion of the wheels in the vehicles. $\overline{4}$.
- The motion of minute and hour hand of a watch. 5.

Angular displacement: The angle traced by a particle is called angular displacement.

$$
\theta = \frac{\text{arc}}{\text{radius}} \text{ or } s = r\theta
$$

Angular velocity: Change in angular position with time is called angular velocity.

$$
\omega = \frac{d\theta}{dt}
$$

Angular acceleration: The rate of change of angular velocity is called angular acceleration.

 $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

Definition of Torque and Angular Momentum 4.1.2

Torque Α.

It is given as the product of force and the perpendicular distance of force from the axis of rotation. Let a force F acting on a point P of a rigid body and the axis of rotation passes through origin O. \vec{r} is the position vector of point P, shown in fig. (4.2) then rotation of the body due to the torque exerted by a force F is given as

Torque = $F \times$ (perpendicular distance of the line of action of a force from axis of rotation)

OM is perpendicular drawn on the line of action of a force from origin O.

 $\sin \theta = \frac{OM}{OP}$ or $OM = OP \sin \theta$ or $OM = r \sin \theta$ In $\triangle OMP$

 τ = F(OM) (OM is called lever arm or moment arm of torque)

So, magnitude of torque $\tau = rF \sin \theta$

Vector form of torque $\vec{\tau} = \vec{r} \times \vec{F}$

S.I unit of torque is N-m and dimensions are ML²T⁻²

Torque is also called the moment of force. The direction of torque given by the right-hand thumb rule because it is the vector product of \vec{r} and F .

Case (1) if $\theta = 0^{\circ}$ or 180° thus, $\sin \theta = 0$

 $\tau = 0$ line of action of force passes through the origin.

Case (2) if $\theta = 90^\circ$ thus, $sin90^\circ = 1$ thus, $\tau = rF$

Examples

- See-saw is an example of torque. One person sitting on one end of the see-saw and another on $1.$ the other end, which is heavier than the other. The heavier person sitting closer to pivot decreases their torque because the length of the moment arm will be shorter than that of a lighter person. With a smaller length of the moment arms of heavier person, smaller is the torque to allowing a lighter person to lift heavier ones.
- A longer handle of the water pump decreases the amount of force applied by human to get $2.$ work done. When larger the moment arm lesser force is required $\vec{\tau} = \vec{r} \times \vec{F}$.

Angular momentum and its examples В.

It is defined as the product of linear momentum and the perpendicular distance of linear momentum from the axis of rotation. Let linear momentum \vec{p} is applied at point P of a rigid body and the position vector of point P is \vec{r} from origin O as shown in fig. (4.3). Angular momentum of the particle about O is

given as $L = p$ (OM)

(OM is perpendicular drawn from origin O to the line of action of linear momentum)

In \triangle OMP, $\sin \theta = \frac{OM}{OP}$ or OM = OPsin θ or OM = rsin θ L = $p(r \sin \theta)$ = mvr sin θ (Linear momentum p = mv) Vector form of angular momentum is given as $L = \vec{r} \times \vec{p}$ Direction of \overline{L} is given by the right-hand thumb rule. S.I unit of angular momentum is kgm^2s^{-1} or J-s, and dimensional formula of it is ML^2T^{-1} Case 1. If $\theta = 0^\circ \sin 0 = 0$ and $\vec{L} = 0$ Case 2. If $\theta = 90^{\circ}$ thus $\sin 90^{\circ} = 1$, thus, $L = mvr$

Fig. 4.3: Angular momentum

Examples

- $1.$ Orbital angular momentum of the earth due to its revolution about the sun.
- $2.$ Spin angular momentum of the earth due to its daily rotation Axis of rotation about its axis.
- $\overline{3}$. Spin angular momentum of a rotation chair about its axis.

$4.1.3$ **Moment of Inertia**

Moment of inertia (a)

The moment of inertia of a particle is defined as the product of mass of the particle and the square of the distance of the particle from the axis of rotation.

 $I = mr^2$

Fig. 4.4: Moment of inertia

 $...(1)$

If a body contains n-particles as shown in fig. (4.4) then moment of inertia of the body is given as

 $I = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2 = \sum_{i=1}^n m_i r_i^2$

 r_1 , r_2 , r_3 ---------- r_n are distances of particles from axis of rotation respectively.

S.I unit of moment of inertia is kgm² and its dimensional formula is $ML²T⁰$

Physical significance of (M.I.) (b)

A moment of inertia is a property of a body due to which it opposes the change in its rotational motion. It plays the same role as the mass in translational motion.

Moment of inertia depends on,

(1) Mass of the body.

(2) Distribution of mass (or distance of the particle from the axis of rotation).

(c) Relation of torque and moment of inertia

We know that torque acting on a body given as $\vec{\tau} = \vec{r} \times \vec{F}$

If position vector \vec{r} of the body is perpendicular to \vec{F} then $\theta = 90^{\circ}$ or sin $90^{\circ} = 1$

 $\tau = rF$

A particle of mass m rotated about axis of rotation in circle of radius r then tangential force on the particle is $F = ma = mr\alpha$ (a = r α) ... (2)

Put tangential force $F = mr\alpha$ in equation (1)

 $\tau = r(mr\alpha) = mr^2\alpha$

If the body having n-particles then net torque on the body can be obtained by summing over all the particles torque exerted due to all forces acting on the particles of the body.

 $\tau_{\text{net}} = \sum_{i=1}^{n} m_i r_i^2 \alpha = I \alpha$

Here, I = $\sum_{i=1}^{n} m_i r_i^2$ is the moment of inertia of the body about the axis of rotation and net torque of the body $\vec{\tau}_{\text{net}} = \Sigma \vec{r}_i \times \vec{F}_i$

Relation of angular momentum and moment of inertia (d)

We know that the angular momentum of a body is given as $\vec{L} = \vec{r} \times \vec{p}$

If position vector \vec{r} of the body is perpendicular to \vec{p} then $\theta = 90^{\circ}$ or sin 90° = 1, Thus, L = rp

A particle of mass m rotated about the axis of rotation in a circle of radius r, then angular momentum

 $L = rp = mvr = mr^2\omega$ (linear momentum $p = mv, v = r\omega$)

If the body having n-particles, then the total angular momentum of the body can be obtained by summing the angular momentum of all the particles due to the linear momentum of all the particles of the body.

$$
L_{net} = \sum_{i=1}^{n} m_i r_i^2 \omega = I \omega
$$

 $I = \sum_{i=1}^{n} m_i r_i^2$ here, I is the moment of inertia of the body about the axis of rotation.

Total angular momentum $\vec{L} = \sum_{i=1}^{n} \vec{r}_i \times \vec{p}_i$

Conservation of Angular Momentum and its Applications $4.1.4$

Conservation of angular momentum (a)

We know that the angular momentum of a body is given as

$$
\vec{L} = \sum_{i=1}^{n} \vec{r}_i \times \vec{p}_i
$$

By differentiating of angular momentum \vec{L} with respect to time, t

$$
\begin{split} \frac{d\vec{L}}{dt}=&\frac{d\sum_{i=1}^{n}\vec{r}_{i}\times\vec{p}_{i}}{dt}=\Sigma_{i}^{n}\bigg[\frac{d\vec{r}_{i}}{dt}\times\vec{p}_{i}+\vec{r}_{i}\times\frac{d\vec{p}_{i}}{dt}\bigg]=\Sigma_{i}^{n}\bigg[\vec{v}_{i}\times m\vec{v}_{i}+\vec{r}_{i}\times\vec{r}_{i}\,\bigg]=\Sigma_{i}^{n}\bigg[0+\vec{r}_{i}\times\vec{r}_{i}\,\bigg]=\Sigma_{i}^{n}\bigg[\vec{r}_{i}\times\vec{r}_{i}\,\bigg]=\vec{\tau}_{net} \\ \frac{d\vec{L}}{dt}=&\vec{\tau}_{net} \end{split}
$$

 $\vec{\tau}_{net}$ is the total or net torque on the body due to all external forces applied to it.

If
$$
\vec{\tau}_{net} = 0
$$
 then, $\frac{d\vec{L}}{dt} = 0$ thus, $L = I\omega = \text{constant or } I_1\omega_1 = I_2\omega_2$

If net external torque on the body is zero (or no torque applied) then the total angular momentum of the body remains constant. This concept is called the principle of conservation of angular momentum.

Examples of conservation of angular momentum (b)

1. Example of the turntable: When a person stands on a turntable with outstretched arms and holds weight in each hand, the turntable revolves with certain angular momentum. Now if he pulls the weight in, towards his body, there is a sudden increase in the angular velocity. This happens due to the moment of inertia of the man decreases when he pulls the weight inward. Here, no external torque exerted so by the conservation of angular momentum while the moment of inertia decreases, angular velocity must be increased so that angular momentum $(I_1 \omega_1 = I_2 \omega_2)$ remains constant.

2. Similarly, Ballet dancers, divers are using the principle of conservation of angular momentum.

4.1.5 **Radius of Gyration**

Moment of inertia of a body is given as $I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$ --------- $\sum_{i=1}^{n} m_i r_i^2$

Moment of inertia of the body can be written as $I = M \times K^2$

Here M is the total mass of the body and K is the radius of gyration.

$$
MK^{2} = \sum_{i=1}^{n} m_{i} r_{i}^{2}
$$
 thus, the radius of gyration $K = \sqrt{\frac{\sum_{i=1}^{n} m_{i} r_{i}^{2}}{M}}$

The radius of gyration K of a rotating body is equal to the radial distance from the axis of rotation, the square of which multiplied by the total mass of the body gives the moment of inertia of that body.

Theorem of Parallel and Perpendicular Axes (Statement only) 4.1.6

Perpendicular axes theorem Α.

This theorem states that the moment of inertia of a lamina about an axis perpendicular to its plain (I_{α}) is equal to the sum of the moment of inertia of the lamina about two mutually perpendicular axes $(I_{\mathbf{v}})$ and I_v) lie in its plain and intersecting at a point where the perpendicular axis passes.

Thus, $I_z = I_x + I_y$

Fig. 4.5: Perpendicular axes theorem

В. Parallel axes theorem of M.I.

This theorem states that the moment of inertia I_{AB} of a body about any given axis (AB) is equal to the sum of moment of inertia about a parallel axis passes through the Centre of mass(C) of the body and the product of the mass M of the body and the square of the distance 'd' between the two parallel axes.

Thus, $I_{AB} = I_C + Md^2$

Fig. 4.6: Parallel axes theorem

4.1.7 **Moment of Inertia of the Following Bodies**

(a) Rod: (1) M.I. about an axis of rotation passes through the centre and perpendicular to the length

$$
I = M\left(\frac{l^2}{12} + \frac{r^2}{4}\right)
$$
 M = mass, l = length, r = radius If r<\frac{Ml^2}{12}

(2) M.I about an axis of rotation passes through one end and perpendicular to the length of the rod I'= $\frac{Ml^2}{3}$

(3) M.I about an axis of the rod $I'' = \frac{1}{2} M r^2$

 (b) *Disc:*

(1) M.I about axis passes through centre and perpendicular to plain = $\frac{1}{2}$ Mr²

(2) M.I about axis passes through centre parallel to plain or lies in-plain or diameter = $\frac{1}{4}$ Mr²

- (3) M.I about an axis is tangent and perpendicular to plain = $\frac{1}{2}Mr^2 + Mr^2 = \frac{3}{2}Mr^2$
- (4) M.I about the tangent, lies in plain = $\frac{1}{4}$ Mr² + Mr² = $\frac{5}{4}$ Mr²
- (c) Ring:
	- (1) M.I about axis passes through centre and perpendicular to the plain $I = Mr^2$
	- (2) M.I about axis passes through centre, lies in the plain or diameter I = $\frac{1}{2}$ Mr²
	- (3) M.I about an axis as tangent and lies in plain = $\frac{1}{2}Mr^2 + Mr^2 = \frac{3}{2}Mr^2$
	- (4) M.I about tangent and perpendicular to plain $I = Mr^2 + Mr^2 = 2Mr^2$
- (d) Sphere (solid):
	- (1) M.I about axis passes through the centre or about diameter I = $\frac{2}{5}$ Mr²

(2) M.I about tangent =
$$
\frac{2}{5}
$$
 Mr² + Mr² = $\frac{7}{5}$ Mr²

(e) Hollow sphere:

(1) M.I about axis passes through the centre or about diameter I = $\frac{2}{5}M\left(\frac{r_2^5-r_1^5}{r_2^3-r_1^3}\right)$ $M =$ mass, $r_2 =$ outer radius and $r_1 =$ inner radius

(f) Spherical shell: M.I about axis passes through centre or diameter I= $\frac{2}{3}$ Mr²

Applications (Real-Life/Industrial)

- The hand flour grinder is provided with the handle near its rim to increases the moment arm 1. (or distance between the line of action of force and the pivoted point). So, torque increases and grinding of flour can be done with less effort.
- The angular momentum of planets also remains conserved when the planets revolve around 2. the sun.
- Ice skaters use conservation of angular momentum to change angular velocity by changing its $3.$ moment of inertia.

Case-Study (Environmental/Sustainability/Social/Ethical Issues)

When a cat falls to the ground from a height, it stretches its body along with the tail. The moment of inertia of the cat remains high and no external torque is applied. So, total angular momentum remains conserved because the cat follows the principle of conservation of linear momentum.

L = I ω = constant (because here $\tau = 0$)

 $I_1\omega_1 = I_2\omega_2$, $I_1 < I_2$, $\omega_1 > \omega_2$

The cat stretches out its leg so, the moment of inertia increases therefore angular velocity decreases. Thus, the cat lands on its feet on the ground safely (or without injury).

Create Inquisitiveness and Curiosity

- 1. Explain the direction of the centripetal force while the body is rotated clockwise and anticlockwise.
- 2. A body can be rotated while torque is zero. Explain that situation
- 3. A ring and a disc of different material have equal mass and equal radii. Explain moment of inertia of the ring is greater than that of the disc.

Solved Problems

Problem-1: A uniform ring of mass 200g and radius of 20cm is rotated about its diameter at an angular speed of 10rad/s. Find (1) moment of inertia of ring. (2) angular momentum about the given axis. Solution:

(1) M.I of ring I = $\frac{1}{2}MR^2 = \frac{1}{2} \times 0.2 \times 0.2 \times 0.2 = 4 \times 10^{-3}$ kgm²

(2) angular momentum L = $I\omega = 4 \times 10^{-3} \times 10 = 4 \times 10^{-2}$ Js

Problem-2: Find the radius of gyration of a hollow uniform sphere of the radius R about its tangent.

Solution: Moment of inertia of a hollow sphere about its diameter $I_{dia} = \frac{2}{3}MR^2$

So, M.I of a hollow sphere about its tangent $I = I_{dia} + MR^2$ (by parallel axis theorem)

$$
I = \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2
$$

If K is the radius of gyration, then, $MK^2 = \frac{5}{3}MR^2$ or $K = R\sqrt{\frac{5}{3}}$

Problem-3: Find the torque of a force $\vec{F} = 2\vec{i} + 3\vec{j}$ about origin when it acted upon a particle at point (1,1,0)

Solution: Torque $\vec{\tau} = \vec{r} \times \vec{F}$, the position vector of the particle $\vec{r} = \vec{i} + \vec{j} + 0\vec{k} = \vec{i} + \vec{j}$

 $\vec{\tau} = (\vec{i} + \vec{i}) \times (2\vec{i} + 3\vec{j}) = 2(\vec{i} \times \vec{i}) + 3(\vec{i} \times \vec{j}) + 2(\vec{i} \times \vec{i}) + 3(\vec{j} \times \vec{j}) = 3\vec{k} - 2\vec{k} = \vec{k}$ Nm

Problem-4: Find the moment of inertia of disc (1) about its diameter and (2) tangent perpendicular to its plain.

Solution:

(1) $I_z = I_x + I_y = 2I_x = 2I_y$ or $I_{dia} = I_x = I_y = \frac{1}{2}I_z = \frac{1}{2}(\frac{1}{2}MR^2) = \frac{1}{4}MR^2$ (by perpendicular axes theorem)

(2) M.I of disc about tangent perpendicular to plain $I_t = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$ (by parallel axes theorem)

UNIT SUMMARY

- All the particles of a body transfer from one point to another with the same velocity then \bullet motion is called translational motion and if the body rotated about an axis, then motion is called rotational motion.
- Moment of force is called torque $(\vec{\tau} = \vec{r} \times \vec{F})$ and moment of linear momentum is called angular momentum $\vec{L} = \vec{r} \times \vec{p}$.
- If net torque applied on a body is zero then the angular momentum of the body remains \bullet conserved $I_1\omega_1 = I_1\omega_2$. This fact is called the principle of conservation of angular momentum.
- The product of mass and distance from the axis of rotation is called the moment of inertia \bullet $I = Mr^2$.
- $I_z = I_x + I_y$ is the perpendicular axes theorem and $I_{AB} = I_C + Md^2$ is the parallel axes theorem. \bullet
- The moment of inertia depends on the mass and the distribution of mass. The radius of gyration \bullet is given as $K = \sqrt{\frac{\sum_{i=1}^{n} m_i r_i^2}{M}}$

Moment of inertia of ring I = Mr² and of the disc, I = $\frac{1}{2}$ Mr² about axis passes through Centre \bullet and perpendicular to their plain. M.I of the rod about an axis perpendicular to the length and passes through midpoint I = $\frac{1}{12}$ ML² and M.I. of the solid sphere about its diameter I = $\frac{2}{5}$ MR².

EXERCISES

Objective Questions (A)

 $[LOD1]$

3. Moment of inertia is a vector quantity

Answers of Objective Questions А.

 $I_x = 0$, $I_y = \frac{1}{3}ML^2$, $I_z = \frac{1}{3}ML^2$, total $I = 0 + \frac{1}{3}ML^2 + \frac{1}{3}ML^2 = \frac{2}{3}ML^2$ $A1.$

Subjective Questions (B)

- 4.1 (a) State principle of conservation of angular momentum.
	- (b) State parallel and perpendicular axes theorems of the moment of inertia.
	- (c) Define the radius of gyration.
- A particle of mass 3kg is moving with velocity $(3\vec{i}+4\vec{j})$ m/s. Find angular momentum of the 4.2 particle about origin when it is situated at the point $(2, 2, 0)$. $[LOD2]$
- A mass of 4kg rotates on a circle of radius 1.0m with an angular velocity of 40rads⁻¹. If the radius of 4.3 the path becomes 2.0m. Then, find angular velocity in the increased radius of the circle. [LOD2]
- The angular momentum of a rotated body is 30Js and frequency (or rate of rotation) is 30rev/s. 4.4 Find the moment of inertia of the body. [LOD3]
- 4.5 A ring of mass 40g has a radius of 20cm. Find the moment of inertia of ring about (1) an axis passes through Centre and perpendicular to its plain (2) tangent and perpendicular to plane of the ring. [LOD2]
- An external torque of 200N-m applied to a wheel to rotate it. The moment of inertia of the wheel 4.6 about an axis of rotation is 25kgm². Find (1) angular acceleration (2) angular velocity after 3s of starting from rest. [LOD2]
- Moment of inertia of a body is 2kgm². Calculate applying torque which will be produced angular 4.7 acceleration 5rads⁻². [LOD1]
- 4.8 Moment of inertia of ring about an axis passes through its centre and perpendicular to its plane is MR². Find the moment of inertia about its diameter. $[LOD2]$

B. Answers of Subjective Questions

 $\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v}) = 3(2 \vec{i} \times 2 \vec{i}) \times (3 \vec{i} \times 4 \vec{j}) = 6 \vec{k}$ kgm²s⁻¹ $A4.2$ $I_1\omega_1 = I_2\omega_2$ or $I_1 = mr_1^2$, $I_2 = mr_2^2$, $\omega_2 = 10 \text{rad/s}$ A4.3 A4.4 $L = I\omega = I2\pi n$ or I = 0.16 kgm² A4.5 $I = MR^2 = 1.6 \times 10^{-3}$ kgm² and $I = MR^2 + MR^2 = 3.2 \times 10^{-3}$ kgm² A4.6 $\tau = I\alpha$, $\alpha = 8$ rad/s² and $\omega = \omega_0 + \alpha t = 0 + 8 \times 3 = 24$ rad/s A4.7 $\tau = I\alpha = 2 \times 5 = 10$ Nm A4.8 $I_z = I_x + I_y = 2I_x = 2I_y$ or $I_{dia} = I_x = I_y = \frac{1}{2}I_z = \frac{1}{2}MR^2$

PRACTICALS

To find the moment of inertia of a flywheel. 1.

Practical significance

This experiment will introduce concepts of rotational dynamics. A realistic analysis of the motion of the flywheel can be made in this experiment. But it will assume that the total frictional torque on the rotating flywheel remains constant. Apply rotational dynamics and will be determined measurement errors.

Relevant theory

Moment of inertia: It is the property of a body due to which body opposes the change in its state of rest or uniform rotational motion. The moment of inertia depends on the mass and distribution of mass of the body about the axis of rotation.

Let n_1 are the number of turns of string winding on the axle and 'r' is the radius of the axle. Suppose mass 'm' is attached with the free end of the string and this mass falls through height 'h' after release from the axle. By the law of conservation of energy potential energy = mgh of mass 'm' is converted into three forms as given below

The potential energy of mass 'm' = KE of mass 'm' + Rotational KE of wheel + Work done against frictional force

$$
mgh = \frac{1}{2} mv^{2} + \frac{1}{2} I\omega^{2} + n_{1}W_{f}
$$
...(1)

Here I is moment of inertia and W_f is work done against the force of friction of axle, $(v = r\omega)$, rotational K.E. of the wheel is equal to work done which is done by wheel against friction before comes to rest after $n₂$ rotations,

thus,
$$
\frac{1}{2} \text{I}\omega^2 = n_2 W_f
$$
 or $W_f = \frac{\text{I}\omega^2}{2n_2}$ put W_f and v in equation (1)
\n
$$
\text{mgh} = \frac{m\omega^2}{2r^2} + \frac{1}{2} \text{I}\omega^2 + n_1 \frac{\text{I}\omega^2}{2n_2}
$$
\nor
$$
\text{I} = \frac{\left(2 \text{mgh} - \frac{\text{m}\omega^2}{r^2}\right) n_2}{\omega^2 (n_1 + n_2)} \text{ but } \frac{\text{m}\omega^2}{r^2} \text{ is negligible due to a small value of } \omega
$$
\nSo, the moment of inertia of flywheel I = $\frac{2mghn_2}{\omega^2 (n_1 + n_2)}$...(2)

If the flywheel takes time 't' for n , number of rotations, the average angular velocity of the wheel is given as $\omega_{avg} = \frac{2\pi n_2}{t}$

If the initial angular velocity of wheel is ω when mass 'm' touched the ground and final velocity of wheel is zero when finally rotations of wheel stopped, so, the average angular velocity of the wheel is

$$
\omega_{\text{avg}} = \frac{\omega + 0}{2} = \frac{\omega}{2}, \text{ thus } \frac{\omega}{2} = \frac{2\pi n_2}{t} \text{ or } \omega = \frac{4\pi n_2}{t}
$$

Put the value of in equation (2) then, I = $\frac{\text{mght}^2}{8\pi^2 n_2 (n_1 + n_2)}$...(3)

Here height 'h' through which mass 'm' falls until string detached is given as

h = circumference of axle \times number of turns of string wrapped on axle = (2 πr) n₁

Put 'h' in equation (3),

thus, the moment of inertia of flywheel I = $\frac{mgrn_1t^2}{4\pi n_2(n_1+n_2)}$

Here m = mass attached with another end of a string, r = radius of the axle, n_1 = number of turns of string wrapped on the axle, n_2 = number of rotations of the flywheel before it stops, t = time measured in stopwatch and count of n₂ both start when string detached and till rotations of flywheel stops.

Practical outcomes (PrO)

PrO1: Apply the concept of moment of inertia

PrO2: Find the moment of inertia of a flywheel

PrO3: Compare the moment of inertia of the flywheel with its theoretical value of M.I.

Practical setup (drawing/sketch/circuit siagram/work situation)

Fig. 4.7: Flywheel

Resources required

Precautions

- $\mathbf{1}$. There should be no overlapping or a gap between the turns of string wounded on the axle.
- $\overline{2}$. String or thread should be thin in comparison to the diameter of the axle.
- $3.$ Care should be taken that no push is imposed on the flywheel by a person who is taking the reading.
- $\overline{4}$. Don't stand just below the apparatus when released the mass.
- 5. Stopwatch must be handled with care to avoid error in measurement of time.

Suggested procedure

- $\mathbf{1}$. There should be the least friction in the flywheel.
- $2.$ n_1 is the number of turns of string wounded on the axle and keep $n_1 = 8,9$ & 10 for three readings of a mass attached with another end of the string (suggested).
- 3. String or thread should be wound evenly or uniformly over the axle.
- 4. Mark on the circumference of the flywheel so that it will be useful to count the number of rotations of the wheel and can be measure part of the length of circumference in the last incomplete rotation.
- $5.$ The stopwatch should be started just after detaching the loaded string from the axle and stop it when the flywheel stops to rotate. This way time t is measured by a stopwatch.
- The number of rotations n_2 of the flywheel should be counted simultaneously with time 't' 6. measured as above and last incomplete rotation is measured by fraction x/y, where x is the length of the part of the circumference of wheel covered in last incomplete rotation and y is the circumference of the wheel, both length x and y are measured with help of thread and meter scale. (See video, a QR code is given).
- The radius of the axle is measured by vernier caliper, where radius $r = \frac{1}{2}$ (mean diameter) and 7. readings of diameter are equal to the sum of main scale reading (MSR) and product of the coinciding division of vernier and least count (L.C.).
- 8. Slotted weight is used with hanger for mass attached with another end of the string.

Observations and calculations Observation table for the diameter of the axle

Table for the moment of inertia (I)

Calculations

Mean diameter of $axle =$cm

Corrected diameter of axle $d = Mean$ diameter – (\pm zero error of vernier caliper)

The corrected radius of axle
$$
r = \frac{d}{2} = \dots
$$
 cm

The moment of inertia of the flywheel is calculated by formula $I = \frac{mgrt^2n_1}{4\pi n_2(n_1+n_2)}$
Mean value of the moment of inertial of $I = \frac{1}{2}$ Mean value of the moment of inertia of flywheel = kgm^2

Results and/or interpretation

Moment of inertia of flywheel $I =$ kgm²

Conclusions and/or validation

(To be filled by student)

Practical related questions or viva-voce questions

- 1. Define the moment of inertia.
- Define angular velocity and state its relation with frequency (cycle per second). $2.$
- $3.$ Describe a daily life application of flywheel.
- Explain the conversion of potential energy lost by the falling mass in different forms. 4.
- 5. Explain the functions of the flywheel in a machine.

Suggested assessment scheme

(To be filled by teacher)

The given performance indicators should serve as a guideline for assessment regarding process and product-related marks.

* Marks and percentage weightage for product and process assessment will be decided by the teacher.

KNOW MORE

Rotational kinetic energy KE = $\frac{1}{2}$ Iω². We know KE = $\frac{1}{2}$ mv² = $\frac{1}{2}$ m(ωr)² $=\frac{1}{2}mr^2\omega^2=\frac{1}{2}I\omega^2$ (v = r ω). Work done in rotational motion W = Fs = mas = m(α r) (r θ) = mr² α θ = I $\alpha\theta$ = $\tau\theta$ where (a = α r), (s = r θ), $(I = mr^2)$, $(\tau = I\alpha)$. Power = work/time = $\tau\theta/t = \tau\omega$. Symbols have the usual meaning.

Innovative Practical/Projects/Activities

- (1) To find the moment of inertia of the rod.
- (2) To find the moment of inertia of the ring.

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Suggested Learning Resources for Practical

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DDD

5 Properties of matter

UNIT SPECIFICS

This unit focusses on the following aspects of basic physics:

- Concepts of elasticity in solids, stress and strain, moduli of elasticity, Hooke's law, the significance of stress-strain curve
- Concepts of pressure, atmospheric pressure, gauge pressure, absolute pressure, Fortin's Barometer and its applications
- Surface tension and its applications, cohesive and adhesive forces, angle of contact, ascent Formula, the effect of temperature and impurity on surface tension
- Viscosity and its applications in hydraulics, coefficient of viscosity, terminal velocity, Stokes' law and effect of temperature on viscosity
- Concepts of hydrodynamics, fluid motion, streamline and turbulent flow, Reynolds number equation of continuity, Bernoulli's theorem and its applications.

RATIONALE

Stress, strain and modulus of elasticity are classified. Statement of Hooke's law and significance of stressstrain curve explained. The concept of pressure, method and device of its measurement are described such as Fortin's barometer. The role of cohesive and adhesive force in surface tension is justified. Uses of surface tension and effect of temperature, impurity is given. Stokes' law of viscosity and its use to find terminal velocity is described. Effect of temperature on viscosity and applications of viscosity in the hydraulic system are given. Types of fluid motion, Bernoulli's equation, Reynold's number and continuity equation are also included in the text.

PRE-REQUISITES

Physics-Basics of Force, motion and work Mathematics-Basics of algebra Other-Basic knowledge of computer

UNIT OUTCOMES

After learning this unit students able to

- U5-O1: State types of stress, strain and modulus of elasticity and Hooke's law. Give significance of the stress-strain curve.
- U5-O2: Relate atmospheric pressure with a gauge and absolute pressure and use Fortin's Barometer in the measurement of atmospheric pressure.
- U5-O3: Distinguish between cohesive and adhesive forces. Apply surface tension in engineering problems and explain the effect of temperature and impurity on surface tension.
- U5-O4: State Stokes' law and find the terminal velocity of the hydraulic system. Determine the effect of the temperature on viscosity.
- U5-O5: Distinguish between streamline and turbulent flow of fluids. Explain Reynolds number and state Bernoulli's theorem and apply it in daily life.

Mapping of unit outcomes with course outcomes:

5.1 ELASTICITY

Interesting facts

The property of a body due to which after removal of an externally applied force it regains its original configuration (shape and size) is called elasticity and body is known as an elastic body. If after removal of external force body does not regain its initial configuration then it is called a plastic body. Deforming force: It is an externally applied force under which dimensions of a body changes. Intermolecular forces are experienced between molecules of each material when an external force applied to a body these intermolecular forces get changed and restoring forces are developed on the molecules. **Restoring force:** It is the force produced inside the body which is equal in magnitude but opposite in direction of external force applied on it. Restoring force brings the molecules in their original position after deforming force is removed. So, intermolecular forces are responsible for the property of elasticity of the materials.

5.1.1 Definition of stress and strain

(a) Stress

When external deforming force is applied then equal and opposite force appears inside the body which tries or tends to recover the original size or shape of the body. Restoring force per unit area produced inside the body due to externally applied force is called stress.

S.I unit of stress is N/m^2 or pascal (Pa). Dimensional formula of stress is $ML^{-1}T^{-2}$. It is a tensor quantity because it has different values in different directions.

There are three kinds of stress depending on the force that acts on the body in which manner or way.

- 1. Longitudinal or normal stress: When the external deforming force acting along the length of the body (wire or rod) then that force per unit area is called longitudinal stress.
	- **a.** Tensile stress: The perpendicular force per unit area of cross-section which producing elongation of the wire is called tensile stress. When equal forces of magnitude F are applied perpendicularly to the cross-sectional area of the ends of the wire or rod away from it as

shown in (Fig.5.1), then stress is given as tensile stress = $\frac{F}{A}$

b. Compressive stress: The perpendicular force per unit area of the cross-section that producing contraction of the body is called compressive stress. It is shown in (Fig.5.2) and

given as compressive stress = $\frac{F}{A}$

Fig. 5.1

2. Volumetric stress: When the same pressure or force per unit area is acting at each point of the body normally then force per unit area is called volumetric stress or also called pressure. When pressure on a body is increased from P to $(P + \Delta P)$ the volume of the body decreases from V to $(V - \Delta V)$ as shown in (Fig.5.3). Change in pressure ΔP is the force per unit area due to which volume of the body changes.

 $\frac{\mathsf{F}}{\mathsf{A}}$

Fig. 5.2

Volumetric stress = change in pressure = $\Delta P = \frac{F}{A}$

The shape of the body may or may not change depending upon the the symmetry of the body. 3. Shear stress: When force is applied tangentially to the surface of a body due to which its shape changes then force per unit area acts on the surface is called shear stress. If the tangential force

exerted is F, then, shear stress = $\frac{F}{A}$.

(b) Strain

The ratio of change in dimension (length, volume or shape) to the original dimension of the body is Change in dimension

called strain. i.e., strain = $\frac{c}{\text{Original dimension}}$

Strain is a unitless and dimensionless quantity.

Types of strain

There are three types of strain associated with each type of stress defined above.

1. Longitudinal strain: when deforming force applied on the rod or wire of length L and length changes by L either increases from L to $(L + \Delta L)$ or decreases from L to $(L - \Delta L)$, the ratio of change in length to original length is called the longitudinal strain

 $\frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta L}{L}$ $longitudinal strain =$

2. Volumetric strain: When volumetric stress is applied to a body and its volume changes, then the ratio of change in volume to original volume is called volumetric strain. If the original volume of a body is V and V + Δ V or V - Δ V is the volume after pressure exerted. Change in volume Then it is given as volumetric strain = $\frac{1}{\text{Original volume}}$

3. Shear strain: When shear stress is applied then the shape of the body changes. Suppose tangential force F is applied along the upper

surface of a body then the shape of the cross-section changes from a rectangle to a parallelogram. Lower edge AD remains fixed and upper edge BC displaced by x as shown in (Fig.5.7). Then shear strain is defined as the ratio of displacement of the upper edge to its perpendicular distance from the fixed edge.

Shear strain = $\frac{BB'}{AB}$ (Here $\tan \theta \approx \theta = \frac{BB'}{AB} = \frac{x}{L}$) Thus, shear strain = $\frac{x}{1}$

Hooke's Law and Modulus of Elasticity $5.1.2$

Hooke's law

This law states that within an elastic limit, the stress in a body is proportional to the strain in it.

i.e., stress ∞ strain

or stress = E strain (Here E is constant and known as modulus of elasticity)

$$
E = \frac{Stress}{Strain}
$$

Modulus of elasticity

The ratio of stress to strain is called the modulus of elasticity. S.I unit of modulus of elasticity is N/m^2 or pascal (Pa). It is dependent on the nature of the body and independent of the length and the volume of that body.

$5.1.3$ **Types of Moduli of Elasticity**

There are three types of moduli of elasticity related to three types of stress and strain correspondingly.

Young's modulus of elasticity (Y) $\mathbf{1}$

When force F is applied on a rod of the area of cross-section A then longitudinal stress $\left(\frac{F}{A}\right)$ is developed. Let the original length of the rod is L and due to the stress length changes by ΔL . Then longitudinal strain = $\frac{\Delta L}{L}$.

By Hooke's law, for a small value of strain young's modulus is defined as the ratio of longitudinal Longitudinal stress

stress to longitudinal strain. So young s modulus $Y = \frac{Q}{\text{Longitudinal strain}}$

$$
Y = \frac{F_A}{\Delta L/L} = \frac{FL}{A \Delta L}
$$

Bulk modulus $2₁$

The ratio of volumetric stress to the volumetric strain of a body is known as bulk modulus. So, bulk

modulus B = $\frac{\text{Volumetric stress}}{\text{Volumetric strain}} = \frac{\Delta P}{-\Delta V/V} = \frac{-V(\Delta P)}{\Delta V}$

(-) sign indicates that pressure increases by (ΔP) then volume decreases by (ΔV)

Compressibility (K): K is reciprocal of bulk modulus B.

 $K = 1/B$

S.I unit of compressibility is m^2/N or 1/pascal (Pa⁻¹)

3. Shear modulus or modulus of rigidity

The ratio of shear stress to shear strain is called the shear modulus. It is also known as the modulus of rigidity or torsional modulus. It is denoted by η , so,

Modulus of rigidity $\eta = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F/A}{x/1} = \frac{FL}{Ax}$

$5.1.4$ **Significance of Stress-Strain Curve**

To study the behaviour of a wire of uniform area of cross-section under the influence of applied force it is suspended from a rigid base and weight is slowly increased at its free end then the length of wire also increases. The curve between stress and strain is shown in (Fig.5.8). In this graph, stress is proportional to strain up to point A, and OA is a straight line, this point A is called proportional limit. Hooke's law is obeyed up to point A.

Now strain further increases from A to B with the increase in stress, the stress is not proportional to strain but the strain increases more rapidly than the stress and Hooke's law is not obeyed. Even between points A and B after removal of the deforming force, the wire comes to its original length. Thus, elastic behaviour is shown by wire up to point B. The endpoint B of this OB part of the curve is called the elastic limit or yield point.

Beyond the elastic limit B, say point C, even after the removal of deforming force the wire does not regain its original length. Some permanent increase in length remains in the wire that shown in fig.

(5.8) by OY and it is called permanent set. Now wire behaves as a plastic material. Further increase of deforming force or weight beyond point C, strain increases more rapidly with a small increase in stress and wire breaks at point D which is called fracture point and stress at point D is called breaking stress or ultimate stress. Wire behaves as a plastic material between points B to D.

 $OA = Proportional limit.$

 $D =$ Fracture point

 $OY =$ Permanent set

A material with a small plastic region (BD) is called brittle material and examples are glass, castiron, stone etc. A material of high plastic region (BD) is called ductile material. Ductile materials will bear more deformation before fracture and examples are lead, annealed steel etc.

Applications (Real-life/Industrial)

- 1. The life of a bridge depends on the elastic strength of the material used in it. If the material of low strength is used, the bridge loses its elastic strength with time then the bridge is declared unsafe after long use.
- To determine the maximum height of a mountain. The pressure at the base of the mountain $2.$ pgh is equal to stress. The elastic limit of a mountain is approximately 3×10^8 N/m². The stress must be less than the elastic limits, otherwise, the rock begins to flow.

$$
h < \frac{3 \times 10^8}{\rho g} \quad \text{or } h < 10^4
$$

$$
h \approx 10^4 \text{m} \quad \text{Here density of rock } \rho = 3 \times 10^3 \text{kg/m}^3, g = 9.8 \text{m/s}^2
$$

3. The torsion constant of wire is given as $C = \frac{\eta \pi r^4}{2l}$ where η is the modulus of rigidity, r is radius and l is the length of wire. If the torque τ is required for twisting the wire by an angle θ then τ = C θ and work done in twisting that wire is given as W = ½C θ ².

Case-Study (Environmental/Sustainability/Social/Ethical Issues)

Ultimate tensile strength (UTS) is also called tensile strength or ultimate strength. It is the maximum stress that material can afford or withstand while being stretched or pulled before breaking. In brittle materials, the ultimate tensile strength is close to the yield point, whereas in ductile material the ultimate tensile strength can be higher.

It is determined by tensile test and recording in the engineering. Ultimate strength is the highest point of the stress-strain curve and its unit is the unit of stress.

Create Inquisitiveness and Curiosity

- 1. If a student puts stress on X-axis and strain on Y-axis by mistake then explain that young's modulus is reciprocal of the slope of this curve.
- 2. Discuss that as the temperature increases the modulus of elasticity of a material decrease.

Solved Problems

Problem-1: A stress of 5kg wt./cm² is applied to a wire. Find the percentage increase in its length if young's modulus is 2×10^{11} Pa.

Solution: stress = 5kg wt. /cm² = 5 × 9.8/(10⁻²)² and Strain =
$$
\frac{\Delta L}{L} = \frac{Stress}{Y} = \frac{5 \times 9.8}{2 \times 10^{11}} = 2.45 \times 10^{-6}
$$

% increase in length = $\frac{\Delta L}{L} \times 100 = 2.45 \times 10^{-4}$

Problem-2: A glass cube of side 2 cm has one side fixed when a tangential force equal to 1200N is applied to the opposite face. Find the shearing strain and the lateral displacement of the strained face. The modulus of rigidity of glass is 2.6×10^{10} N/m².

Solution: Stress =
$$
\frac{F}{A} = \eta \frac{x}{L}
$$
 or $x = \frac{FL}{\eta A} = \frac{1200 \times 2 \times 10^{-2}}{4 \times 10^{-4} \times 2.6 \times 10^{10}} = \frac{6}{2.6} \times 10^{-6} = 2.31 \text{ }\mu\text{m}$
and shearing strain = $\frac{x}{L} = \frac{2.31 \times 10^{-6}}{0.02} = 1.16 \times 10^{-4}$

5.2 **PRESSURE**

Interesting Facts

The camel's legs are wide at the bottom, causing the area to be high and the pressure to be low, so the camel can walk easily in the desert. The tip of the needle has a low area which causes high pressure and easily insert into the fabric. To avoid damage, the walls of a dam are kept thicker as depth increases because of more pressure at more depth therefore greater force exerted at large depth.

$5.2.1$ **Definition of Pressure**

Pressure in a static fluid: When a fluid at rest, force is exerted normally to the surface of the walls of the container or on a body dipped in the fluid.

Pressure at a point of a fluid is the average force acting per unit area surrounding that point. If the magnitude of the average force exerted in the fluid is F and area is A. then pressure is defined as

$$
P = \frac{F}{A} \tag{1}
$$

Pressure is a scalar quantity because the direction of pressure is not fixed. S.I unit of pressure is $N/m²$ or also called pascal and abbreviated as (Pa).

Relation of fluid pressure with height

Let two points B and C inside the liquid are situated at vertical height h apart. Suppose area A is containing point B on the upper side and equal area A is containing point C on the lower side such that fluid enclosed in the cylindrical shape of height h and cross-sectional area A as shown in (Fig.5.9). Forces are exerted on each part of fluid normally. Suppose force F_1 is acting vertically upward at point C, force F_2 is acting vertically downward at point B and the weight of the liquid is downwards. Assume pressure at surface B is P_2 and at surface C is P_1 . Then, $F_1 = P_1 A$ $F_2 = P_2 A$

Weight of liquid column W = mass of liquid \times g = mg = (Ah) ρ g

(Here volume of the cylindrical portion of liquid is Ah, the density of the liquid is ρ)

In the vertical equilibrium condition $F_1 = F_2 + W$ or $P_1A = P_2A + Ah$ ρg

 $(P_2 - P_1)$ A = -Ah pg or ΔP = -pgh so, $\Delta P \propto$ -h

(-) sign shows that when we move up in liquid by height h then pressure decreases by value $\rho gh.$

$5.2.2$ **Atmospheric Pressure**

Earth is surrounded by layers of air which forms the atmosphere of the earth. Weight of air column is exerted pressure on the bodies at earth's surface.

Atmospheric pressure is defined as force per unit area exerted by air column on the earth surface.

Let small area A on the earth surface and force exerted by the air on this area is F. Then the

atmospheric pressure $P_{atm} = \frac{F}{A}$

Atmospheric pressure is measured by an instrument which is called the barometer. It devised by Torricelli in about 1644. In a barometer, a glass tube open at one end of sufficient height is filled with mercury. The tube is inverted in a vessel of mercury such that its open end dipped into the vessel. Mercury column settles in the tube as shown in (Fig.5.10).

Atmospheric pressure is equal to the pressure exerted by the height of the mercury column $h = 76$ cm raised in a barometer tube.

Atmospheric pressure P_{atm} = pgh = $13.6 \times 10^3 \times 76 \times 10^{-2} \times 9.8$ 1 atm = 1.013×10^5 Pa ($\rho = 13.6 \times 10^3$ kg/m² density of mercury) Unit of pressure used in the meteorology is called bar and 1 bar = 10^5 Pa, $1 atm = 1.013bar$ One more unit of atmospheric pressure is given that is torr and 1 torr = 1mm of Hg = 133.3Pa, 1 atm = 760 torr

Fig. 5.10: Barometer

Gauge Pressure and Absolute Pressure $5.2.3$

The manometer is a device that is used to find pressure in a closed container of gas. It consists of a tube bent in the form of U (Fig. 5.11) and contains a liquid. One end of the tube is connected to a container of gas and the other end is open to the atmosphere. Point A and B are at the same horizontal level so pressure at A (or pressure of gas) is given as

P = pressure at $A =$ atmospheric pressure at (C) + pressure of a liquid column (BC) of height h

Fig. 5.11: Monometer

 $P = P_{atm} + \rho gh$

This total pressure P is called absolute pressure. ρ is the density of a liquid in a U-tube.

The difference in pressure of gas in the container and atmospheric pressure is called gauge pressure. So, gauge pressure = $P - P_{atm} = \rho gh$

$5.2.4$ **Fortin's Barometer and its Applications**

Accurate measurements of the highest pressure in liquid and gases, often required in the laboratory, are made with a Fortin's barometer. This precision barometer was designed in the late eighteen century by Nicolas Fortin, a French instrument maker.

The tube containing the mercury is protected by enclosing it in a brass tube, the upper part of which is made of glass so that the mercury surface may be seen. Readings are taken by a vernier moving over a millimetre-scale of sufficient length to cover the full range of variation in barometric height. The zero of the scale is at the tip of an ivory pointer fixed to the lower end of the brass tube. The mercury reservoir is a leather bag that can be raised or lowered by a screw as shown in (Fig.5.12). Before taking a reading of the barometer height, the screw is adjusted until the tip of the pointer just touches its image on the surface of the mercury. If the surface of the mercury is dusty, this adjustment can still be made with reasonable accuracy. A Fortin barometer thus measures the height of a column of mercury supported by atmospheric pressure. Pressure due to a column of liquid height 'h' and density ' ρ ' is given by P = ρ hg, where g is the acceleration due to gravity. If variation in the density of mercury

with temperature and the variance in \hat{g} with the position on the earth is neglected, we see that the pressure is proportional to the height of the mercury column. For many purposes, therefore, it is sufficiently accurate to find the pressure in terms of the height of the column i.e., in millimeters of mercury (mm of Hg).

Applications of Fortin's Barometer

1. This barometer is commonly used to measure atmospheric pressure in meteorological stations, laboratories and schools. The advantage of this type of barometer is that it permits the inspection of both free surfaces of mercury whose difference in level has to be measured.

It is also used for the measurement of the altitude of mountains and in weather forecasting. $2.$

Applications (Real-life/Industrial)

- 1. Drinking straw: It is a very thin pipe. When the lower end of it is dipped in a soft drink and we suck at the upper end of a straw, the pressure inside the straw and our mouth reduced. But due to atmospheric pressure which is acting at the surface of the soft drink, pushes the soft drink up to the straw into our mouth.
- 2. Similarly, atmospheric pressure acts in syringe, dropper and rubber sucker.

Case-Study (Environmental/Sustainability/Social/Ethical issues)

Hydrostatic paradox: The area is not being used in the equation of pressure $P = P_{atm} + \rho gh$, so the height of the column of the liquid is important for the calculation of pressure and not the shape of the vessel and the area of the base etc. If according to (Fig.5.13), the vessels A, B and C of three different shapes are connected to a horizontal pipe, then water is filled, its surface remains the same in all the three vessels. The amount of water is different, yet the pressure of water at the same depth is the same.

Fig. 5.13

Create Inquisitiveness and Curiosity

Explain that the pressure in the saltwater of the sea will be higher at the same depth than the fresh water of the lake. Explain how atmospheric pressure decreases when you go at height from the earth's surface. Can gauge pressure be negative?

Solved Problems

Problem-1: (a) Find the pressure on a fish at a depth of 1.5km in seawater of relative density or specific gravity 1.02 (b) Determine gauge pressure.

Solution: (a) Density of water = relative density \times density of water at 4 °C = 1.02 \times 10³ kg/m³

Total or absolute pressure $P = P_{atm} + \rho gh$ $P = (1.013 \times 10^5) + (1.02 \times 10^3 \times 10 \times 1.5 \times 10^3)$ $P = (1.013 \times 10^5) + (1530 \times 10^5) = 1.531 \times 10^8$ Pa

(b) Gauge pressure = $P - P_{atm} = \rho gh = 1.53 \times 10^8 Pa$

5.3 **SURFACE TENSION**

Interesting Facts

 $S = \frac{F}{l}$

- 1. The ends of a glass tube become rounded on heating because when glass is heated it melts to a liquid. The surface of liquid tends to have a minimum area. The area of the surface of a sphere is minimum for a given volume. Hence, melted glass acquires a spherical shape.
- 2. Dancing of camphor on the water: When small pieces of camphor come in contact with pure water surface tension of water decreases. Hence, the camphor is jumped towards the region of higher surface tension. Thus, the camphor pieces dance on the water surface.

Fig. 5.14

3. A small raindrop takes a spherical shape because the surface area of a sphere is the least (or smallest) for a given volume.

To understand the above facts of surface tension. Let consider molecule A inside and B on the surface of the liquid as shown in (Fig. 5.14). At any instant molecule, A experiences force by liquid molecules from all directions hence net force on it is zero. But molecule B resides at the surface experiences force by liquid molecule inside the liquid and by air molecules above the surface. So, the net force on molecule B is not zero. Thus, non-zero force on the molecules on the liquid surface is the cause of surface tension and there is the tendency to decrease the surface area of the liquid surface.

$5.3.1$ **Concept of Surface Tension**

Suppose an imaginary line AB drawn on the liquid as shown in (Fig. 5.15). If force F acts perpendicular to the line AB along the surface on both sides of it and the length of the line is l , then the surface tension of the liquid is defined as

$$
\begin{array}{c}\nF_A \\
\hline\n\end{array}
$$

 $...(1)$

Fig. 5.15

S.I unit of surface tension is N/m and its dimensional formula is $\frac{MLT^{-2}}{L} = ML^{0}T^{-2}$. The free surface of a liquid behaves like a stretched membrane due to surface tension and the surface tends to acquire the minimum surface area.

$5.3.2$ **Cohesive and Adhesive Forces**

The matter is made of molecules and the force acting between molecules of a body is called intermolecular force. These intermolecular forces are divided into two categories:

- 1. Cohesive force: The force of attraction between molecules of the same substance is called cohesive force.
- 2. Adhesive force: The force of attraction between molecules of different substances is called adhesive force.

These intermolecular attraction forces are different from gravitational force and the distance up to which intermolecular forces exert between two molecules is called molecular range it is 10^{-9} m.

Examples: Water molecules experience cohesive force when they come in contact. Adhesive force is experienced between glass and water molecules, chalk and blackboard, ink and paper, cement and brick molecules.

$5.3.3$ **Angle of Contact**

When the free surface of a liquid comes in contact, the surface of liquid near the solid acquires a curved shape. When a glass plate is dipped in water partially the surface near the plate is acquired a concave shape because adhesive force between glass and water is more than cohesive force between water-water molecules. If a glass plate immersed in mercury, the mercury surface is acquired a convex shape or the surface of mercury is depressed near the plate.

The cohesive force between mercury molecules is more than the adhesive force between glass and mercury molecules.

The angle between the tangent at the solid surface and the tangent at the liquid surface at the contact is called the angle of contact. The angle of contact θ are shown in (Fig. 5.16) and (Fig. 5.17)

adhesive force $Fa < F_c$ cohesive force

adhesive force $Fa > F_c$ cohesive force Comparison of pairs of solid and liquid for θ < 90° and θ > 90°

Ascent Formula of Liquid in Capillary Tube $5.3.4$

When a glass tube of very small diameter and open at both ends are dipped in a liquid then either liquid is raised or depressed in this capillary tube. The rise or depression of liquid in a capillary tube is called capillarity.

 $2S\cos\theta$ The formula of height raised in the capillary is given as $h =$ $...(1)$

Here S is the surface tension of the liquid, θ is the angle of contact, r is the radius of the capillary tube, ρ is the density of the liquid. If $\theta = 0^{\circ}$, $\cos 0^{\circ} = 1$

$$
h = \frac{2S}{r\rho g} \quad \text{so } h \propto \frac{1}{r} \tag{2}
$$

Here surface tension S and density ρ of the liquid both are kept constant then the height raised in the capillary tube is inversely proportional to its radius.

Tube of insufficient length: Height raised in a capillary tube is given by equation (1). If the length of the capillary tube is less than the raised height (h) then the liquid surface becomes plane at top of the tube and the liquid does not overflow.

5.3.5 **Applications of Surface Tension**

Daily life example

- 1. The kerosene oil in lanterns and the melted wax in a candle, rise through the capillaries in the cotton wick and burn.
- 2. The nib of the pen is split in the middle due to that a capillary is formed in it. When the nib is dipped in ink, the ink rises in the capillary.
- 3. Soap and detergent decrease the surface tension of water so that water soaks into pores and holes easily.
- 4. A small needle can be floated on the water surface.

Industrial applications

- 1. Surface tension is used in detergent formation so that we can improve cleaning properties.
- 2. It is also used for the characterization of food and packing products.

5.3.6 **Factors Affecting the Surface Tension**

- 1. Temperature: The surface tension of a liquid decreases with rise in temperature because the cohesive force decreases due to an increase in its temperature.
- 2. Effect of contamination: If the contaminant like dust, grease or oil spread over the water surface then surface tension reduces.
- 3. Impurity: If any salt is dissolved completely in a liquid its surface tension increases. If the salt is less soluble or floats on the surface as the pouring of soap or phenol the surface tension of water decreases.

Case-Study (Environmental/Sustainability/Social/Ethical Issues)

The farmer ploughs his fields after rains so that the capillaries formed in the soil by insects are broken and water remains in the lower layers of the soil. If ploughing is not done by the farmer the water of the lower layers will rise through the capillaries in the soil and evaporate. The water of the soil is taken up by the plants so ploughing is necessary for farming.

Create Inquisitiveness and Curiosity

Can you explain why the hot soup is tastier than the cold one? Discuss bigger bubbles that can be formed from soap solution than from the water. Small drops of mercury are spherical while a large one is flat.

Solved Problems

Problem-1: Capillary of internal radius 8mm, is immersed in water. Find height up to which water rises in the capillary. (Surface tension of water S = 70 x 10⁻³N/m, g = 10m/s², $\rho_{\text{water}} = 10^3 \text{kg/m}^2$, $\theta = 0^\circ$ angle of contact).

Solution: The height of liquid in a capillary h = $\frac{2S\cos\theta}{r\rho g} = \frac{2 \times 70 \times 10^{-3}}{8 \times 10^{-3} \times 10^{3} \times 10} = 1.75 \times 10^{-3} = 1.75$ mm

VISCOSITY AND COEFFICIENT OF VISCOSITY 5.4

Interesting Facts

Ketchup or mayonnaise is a thixotropic system, which is characterized by a difference in the viscosity depending on the speed at which shearing force is applied. This means that when it is left undisturbed in a bottle, the ketchup remains thick (high viscosity) and when we shake the ketchup bottle and pour it out, it immediately becomes thin *i.e.*, its viscosity drops.

5.4.1 Viscosity, Coefficient of Viscosity and Terminal Velocity

Viscosity: Fluids (i.e., liquid and gas) exhibit an important physical property, namely viscosity, which is an internal resistance to continuous deformation (or flow) due to tensile or shear stress. It is a tendency to resist the flow of the fluid. For example, honey or syrup is more viscous than water since it experiences more resistance to flow and hence takes more time to flow than water does.

Coefficient of viscosity: Let us consider some fluid in the container. The fluid layer in contact with a bottom surface of the container has the same velocity as that of the bottom surface. For a fixed surface, fluid remains stationary i.e., the velocity of fluid layer in contact with fixed surface is zero. As we move up from bottom layer to top layer in the fluid, the velocity of layers gradually increases. The top layer of fluid in contact with air here, moves with the highest velocity 'v'. There exists a force between any two consecutive layers, which opposes the sliding movement of an upper layer over a lower layer. Such force of friction between two fluid layers due to their relative movement is called as viscous force (F_V) .

When liquid like water, ghee or oil flows in a pipe or a tube, the velocity of a layer in such liquids is maximum along the axis of a pipe and decreases as we move gradually towards the walls of a pipe. Along the cylindrical wall, the velocity of these liquids remains constant.

Fig. 5.20: Velocity distribution of fluid flow in (a) a beaker (b) a pipe

In a laminar fluid flow, two consecutive molecule thick fluid layers move parallelly with varying velocities without mixing over a horizontal surface. For low velocities and high viscosity of a fluid, laminar flow occurs.

At constant temperature, the viscous force (F) acting in a laminar fluid depends on the following factors:

 $F\alpha A$, where F is a viscous force, and A is an area on which viscous force acts.

and
$$
F\alpha \frac{(v_2 - v_1)}{d}
$$
, where $\frac{(v_2 - v_1)}{d}$ is a velocity gradient.
\n
$$
\therefore F\alpha A \frac{(v_2 - v_1)}{d}
$$
\n
$$
\therefore F = \eta A \frac{(v_2 - v_1)}{d}
$$
, where η = coefficient of viscosity
\n
$$
\text{Or } \eta = \frac{\left[\frac{F}{A}\right]}{\left[\frac{(v_2 - v_1)}{d}\right]} = \frac{\text{Shearing stress in fluid}}{\text{Shear rate OR Strain rate OR Velocity gradient}}
$$

This is known as Newton's law of viscosity for incompressible laminar fluid.

The dimensions of the coefficient of viscosity are $[M¹L⁻¹T¹$. Its CGS unit is poise or dyne-s.cm⁻² and, it is measured in N-s.m⁻² in SI unit system. For a highly viscous fluid, the value of 'n' is more and it depends on the temperature of the fluid. For liquids, η decreases as temperature increases, and for gases, it increases with an increase in temperature.

Dynamic (or absolute) viscosity or the coefficient of absolute viscosity and kinematic viscosity or coefficient of kinematic viscosity are two measures of viscosity. Dynamic (or absolute) viscosity is the tangential pressure (or shearing stress) required to move one layer of fluid with respect to another consecutive layer; these layers are at a unit distance apart in the fluid.

Dynamic or absolute viscosity is measured in pascal-seconds (Pa-s) or poiseuille (Pl) or newtonsecond per square meter $(N\text{-}s.m^{-2})$ in SI units. In CGS units, it is measured in poise (P) or centipoise (cP) named after French physicist Jean-Louis Marie Poiseuille.

1 Pa-s = 1 Pl = 10 P

 $1 CP = 0.01 P$, and $1 Pa - s = 1000 CP$

The viscosity of water at room temperature is 1 cP (= 0.01 P or 0.001 Pa-s) which is a benchmark for measuring the viscosity of other liquids. At its boiling point, it drops to 0.038 cP. For ketchup, the viscosity value is about 50,000 to 70,000 cP. Low viscosity chocolate with a viscosity of 40 cP or less is used for thin coating on confectionary items or chocolate fountains, while high viscosity chocolates with a viscosity above 90 cP. For air, viscosity value is 0.019 cP (or 0.000019 Pa-s) and that of motor oil is 0.001 cP (or 1 Pa-s).

Kinematic viscosity is the ratio of a dynamic (or absolute) viscosity to the mass density of a fluid. It is denoted by μ .

$$
\therefore \mu = \frac{\eta}{\rho}
$$

SI Unit of kinematic viscosity is m^2/s and in CGS unit systems, it is stokes (St).

1 St = 1 cm²/s = 10^{-4} m²/s.

Since stokes is a large unit, its $100th$ part i.e., centistokes (cSt) is often used.

Terminal velocity: It is the highest constant velocity with which an object falls through a fluid when the sum of viscous force and force of buoyancy becomes equal to the force of gravity. We denote it by V_{\cdot}

Let us consider a spherical body of mass 'm', radius 'r' and density 'p' falling freely in a meter long glass tube filled with viscous fluid of density ' ρ_f ' and coefficient of viscosity ' η ', experiences three forces: (a) viscous force of the fluid in the downward direction as the body moves in fluid (b) force of buoyancy in the upward direction to keep it buoying (c) gravitational force due to mass of the object in the downward direction. When these three forces acting on a body balance each other, the net force becomes zero and the spherical body travels with a constant velocity or zero acceleration. This constant velocity is known as terminal velocity.

The terminal velocity of a spherical body can be determined by the following formula:

$$
V_t = \frac{2 r^2 g}{9 \eta} (\rho - \rho_f)
$$

If $\rho > \rho_{\rm e}$, then the terminal velocity is positive and the spherical body moves downwards (e.g., steel ball-bearing in water or glycerin) and if $\rho < \rho_0$ then the terminal velocity is negative and the spherical body moves upwards in the fluid (e.g., bubbles in water).

5.4.2 Stokes' Law and Effect of Temperature on Viscosity

Stokes' law: Sir George G. Stokes (1819–1903), an English physicist stated clearly that a small spherical body falling freely in a viscous fluid experiences a viscous drag force (F_v) which is directly proportional to the radius of the spherical body (r), its velocity (V) and coefficient of viscosity (η) of the fluid. He derived the mathematical expression for the viscous force in 1851, which is written as

 $F_V = 6\pi \eta rV$

Stokes' law is valid for a small size body whose dimensions are larger than the distance between fluid molecules i.e., for homogeneous and continuous fluid medium. The body should be perfectly smooth and rigid. The fluid medium should be streamlined or laminar and must extend.

This law does not apply to the particles or bodies that are lighter than the fluid medium (e.g., bubbles rising in soda water or air). Stokes' equation is used for the measurement of the viscosity of fluids and to find the settling of sedimentation in freshwater. If the content of dispersed solids is high, there may be a difference in the real sedimentation rate due to the Brownian motion of molecules of dispersed solids to some extent and the rate measured using Stokes' equation. Capillary tube viscometer, Saybolt viscometer, and rotating viscometers are three types of devices that are used to measure the viscosity of fluids.

Effect of temperature on viscosity: Viscosity is the physical property of a fluid that is used in many engineering fields. Hence, it is quite important to understand the effect of temperature on it.

For liquids, viscosity decreases with an increase in temperature as the cohesive forces binding molecules of liquid reduce and momentum transfer due to the movement of molecules increases. Normally, the cohesive force predominates and liquid molecules remain loosely bound together at normal temperature. This makes liquids at high temperatures less viscous. If a lubricating oil with appropriate viscosity is not used in automobile engines or in refrigerating compressors that can withstand high and low temperatures, then the heat loss due to friction between internal parts reduces their lifespan.

The temperature dependence of viscosity in liquids is given by

$$
\eta = \frac{\eta_0}{(1 + \alpha t + \beta t^2)}
$$
, where η = viscosity of the liquid at 't' °C;

 η_0 = viscosity of the liquid at '0' °C; and α , β are the constants

For gases, viscosity increases with an increase in temperature due to increased momentum transfer rate between molecules as they gain more kinetic energy. The temperature dependence of viscosity in gases is given by

 $\eta = \eta_0 + \alpha t + \beta t^2$

Applications of Viscosity in Hydraulic Systems $5.4.3$

Hydraulic systems are the ones in which forces applied at one point are transmitted to another point through incompressible viscous fluid or medium. For example, for hydraulic brakes, the viscous medium is brake fluid. Any hydraulic system such as hydraulic pumps, motors, cylinders, brakes, etc. used in various applications (e.g., elevator, fishing boats, steering systems, shock absorbers, flight control, and transmission, landing gears in the airplane, earthmoving equipment, tractors, irrigation system, drilling rigs, building and construction machinery, robotic systems, crushers, textile machinery, paper industries, etc.) use incompressible high-density fluid for higher efficiency. This hydraulic fluid can corrode the material of hydraulic systems and therefore selection of material as well as the hydraulic fluid is very important. The viscosity of the hydraulic fluid is affected by a change in temperature due to the inherent function of hydraulics. When temperature increases, its viscosity decreases, and when temperature decreases, its viscosity increases. If its viscosity is high at low temperatures, the hydraulic fluid becomes thicker which can cause cavitation issues, plugged filters, reduction in mechanical efficiency, and overall performance and functionality. If the viscosity of the hydraulic fluid is low at high temperatures, then fluid becomes too thin and it can cause increased friction due to lack of lubrication, increased wear and tear, and internal leakage. Therefore, the hydraulic fluid must have adequate viscosity with respect to temperature changes to maximize lifetime, performance, and reduction in maintenance of components of hydraulic systems.

Applications (Real Life / Industrial)

Viscosity is the most important factor in the selection of lubrication oil used for heavy machinery components. Low viscosity oils are used in car engines. The highly viscous fluid is used as a brake fluid in hydraulic braking. A blood flow through a heart, lungs, and veins depends on its viscosity. In Millikan's oil drop experiment, the viscosity of an oil is used to assess the charge.

High-velocity raindrops have high kinetic energy but when they pass through air, the viscous force due to air reduces their energy, and eventually, raindrops achieve low terminal velocity. This is the reason why we are not hurt by raindrops. Similarly, a parachuter achieves terminal velocity before landing.

Case-Study (Environmental/Sustainability/Social/Ethical Issues)

Various commercial shampoos for different age groups differ from each other because of their rheological properties. One of the main rheological properties is viscosity which correlates with the fluidity and thickness of the shampoo. Shampoos contain 80 wt% water and the remaining substances are surfactants, viscosity modifiers, preservatives, fragrances and colorants. Viscotester rheometers show that for a low shear rate, the shampoos for men and infants have viscosity of about 120 poise, which gives them creamy and more viscous texture, while the shampoos for women have shown viscosity to be about 75 poise, having low viscosity and more fluidity. Hair gels have viscosity of about 960 poise.

Create Inquisitiveness and Curiosity

There is another unit for measuring high kinematic viscosities of fluid-like 'fuel and road oil'. [Hint: The ASTM D 88 test method is prescribed to determine the viscosity of such fluids and the unit includes the portmanteau 'Furol' for 'Fuel and Road Oil.]

Solved Problems

Problem-1: Calculate the terminal velocity of a silicone oil drop of radius 0.3 mm in air. Coefficient of viscosity of air = 1.8×10^{-5} Pa-s; density of silicone oil = 980 kg/m³ and density of air = 1.225 kg/m³. Solution:

Radius of an oil drop = 0.3 mm = 3×10^{-4} m;

Coefficient of viscosity of air, $\eta = 1.8 \times 10^{-5}$ Pa-s;

Density of silicone oil = $\rho = \rho_{\text{oil}} = 980 \text{ kg/m}^3$ and density of air = $\rho_f = \rho_{\text{air}} = 1.225 \text{ kg/m}^3$ Therefore.

Terminal Velocity,
$$
V_t = V_t = \frac{2}{9} \frac{r^2 g}{\eta} (\rho - \rho_f) = \frac{2}{9} \frac{(3 \times 10^{-4})^2}{1.8 \times 10^{-5}} (980 - 1.225) \times 9.8
$$

\n
$$
= 10658 \times 10^{-3} = 10.658 \, \text{m/s}
$$

5.5 **HYDRODYNAMICS**

Interesting Facts

The low viscosity of the blood and high turbulent blood flow causes Anemia in the human body attributed to the generation of Eddy currents in blood vessels. Since the aorta has the largest diameter as compared to the rest of the blood vessels in the body, the blood flow rate is highest which renders turbulence in blood flow and a large Reynolds number. This turbulence allows Eddy currents which cause the blood to hit the blood vessels with a large force. Murmuring due to blood splattering on the heart or the rest of the blood in the vessels is then heard as a distinct sound using a stethoscope. Laminar flow in blood vessels allows the blood cells to adhere to vessel walls and subsequently to the areas of need.

5.5.1 Fluid Motion

Hydrodynamics is a subdiscipline of fluid dynamics and a branch of physics in which the motion of liquids is studied at a macroscopic level in terms of viscosity and mass density. The motion of gases is studied in aerodynamics, another subdiscipline of fluid dynamics. Fluid flows due to unbalanced forces acting on it and it remains in motion as long as it is subjected to these forces. There are different types of fluids depending on how density and viscosity change upon external forces: (1) ideal fluid (2) real fluid (3) incompressible fluid (4) compressible fluid (5) ideal plastic fluid (6) Newtonian fluid (7) non-Newtonian fluid. Ideal fluid is an imaginary incompressible fluid and in reality, it doesn't exist. It doesn't possess viscosity. Real fluid, on the other hand, possesses viscosity. All fluids are real fluids. If the density of the fluid doesn't change when external forces are applied, then such fluid is called an incompressible fluid. For a compressible fluid, density changes with the application of external forces. Ideal plastic fluid is a type of fluid in which shear stress is proportional to velocity gradient and it is more than the yield value. Newtonian fluids obey Newton's law of viscosity, while non-Newtonian fluids do not obey this law. Fluid flow is classified as (1) steady (2) unsteady (3) compressible (4) incompressible (or isochoric) (5) viscous (6) non-viscous (7) rotational (8) irrotational depending on properties of the liquid and how the fluid flows. Fluid flow is considered as steady flow when the velocity of the fluid is constant at any point, while if the velocity of the fluid is not constant at any point in the fluid flow, it is considered unsteady flow. Compressible flow and incompressible flow depend on whether the density of the fluid changes or not with the application of external forces. When the effects of inertia forces, body forces, and pressure gradients in the fluid are balanced by the effects of fluid viscosity, the flow of fluid is called viscous flow. In non-viscous fluid flow, these effects remain unbalanced due to the high velocity of the fluid. Irrotational fluid flow is the one in which fluid particles while moving along the streamlines do not rotate about their axis while in rotational fluid flow, the fluid particles rotate about their axis while moving along the streamlines.

5.5.2 Streamline and Turbulent Flow

If every particle of the fluid flows with a constant velocity at any point of the fluid, the flow is said to be a steady or streamline flow. The fluid particle follows the streamlined path which is a curve whose tangent at any point gives the direction of the fluid velocity at that point. The streamlines never intersect each other in the steady flow of the fluid. In general, laminar flow is a streamline flow, in which the fluid flows in the form of layers of different velocities along well-defined streamlines that do not mix. Streamline flow or laminar flow occurs at a low velocity of the fluid and the exchange of fluid particles does not happen from one layer to another. The viscosity of the fluid dominates over inertia forces in the case of streamline forces therefore, this type of flow is also known as viscous flow. Laminar forces are theoretical.

When the fluid flows with a velocity higher than the critical velocity, fluid particles move erratically, disorderly, and in a random manner. Such fluid flow is known as turbulent flow. The irregular movement of fluid particles from one layer to another layer results in the formation of cross-currents or eddy currents in a turbulent flow. In this type of fluid flow, inertia forces dominate over the viscosity of the fluid. Turbulent flow is a practical and non-viscous flow that occurs at high velocity and low viscosity of the fluid. For example, an airflow just behind the flying airplane or moving bus or train; the flow of water just behind the moving boat or ship are examples of turbulent flow.

5.5.3 Reynolds Number

British physicist Osborne Reynolds derived an empirical formula while observing different flow characteristics of fluids. This formula gives a dimensionless quantity, known as Reynold number (Re) to determine the type of fluid flow pattern while flowing through a tube or a pipe. The Reynolds number (Re) is defined as the ratio of inertia forces to the viscous forces and it can be determined as the ratio of dynamic pressure to shearing stress.

Therefore, Reynolds number, $(Re) = \frac{\rho V D}{\eta} = \frac{VD}{\mu}$ for a circular pipe or a duct, where Re is the Reynolds number, ρ is the density of the fluid, V is the velocity of fluid flow based on the actual area of cross-section of a duct, D is the pipe diameter, and ? is the coefficient of dynamic or absolute viscosity

of fluid and $\mu = \frac{\eta}{\rho}$ is the coefficient of kinematic viscosity.

For a non-circular pipe or a duct, geometrical equivalent diameter is replaced by hydraulic diameter $D = D_h = \frac{4A}{p}$, where A = area of cross-section of a pipe or duct or channel, and p = wetted perimeter

of a pipe or duct or channel. This is also known as 'characteristic length'.

The Reynolds Number can be used to determine the type of fluid flow:

(1) The fluid flow is laminar when $Re < 2300$

(2) The fluid flow is transient when $2300 < Re < 4000$

(3) The fluid flow is turbulent when $Re > 4000$

5.5.4 Equation of Continuity

Let us consider a steady, incompressible (or isochoric), and non-viscous fluid flow in the conduit of a varying area of cross-section. The conduit has a single entry and a single exit. Let A₁, V₁, ρ_1 and A₂, V₂, ρ_2 be the area of cross-section, the velocity of the fluid flow, and density of the fluid at point P and Q of the conduit, respectively.

Then, the volume of the fluid entering per second at $P = A_1 V_1$

Mass of the fluid entering per second at $P = A_1 V_1 \rho_1$

Similarly, the volume of the fluid leaving per second at $Q = A_2 V_2$

mass of the fluid leaving per second at $Q = A_2 V_2 \rho_2$

For an incompressible fluid, density remains the same i.e., it does not vary with the application of external forces. Therefore, $\rho_1 = \rho_2$.

 \therefore A₁V₁ = A₂V₂

 \therefore AV = constant = R = volume flow rate

This is called an equation of continuity for steady flow. It is defined as the product of the area of cross-section of a conduit or a tube and the velocity of the incompressible fluid at any given point along the conduit or a tube is constant. This constant is also known as volume flow rate. This equation states that if the area of cross-section of the conduit becomes larger, the velocity of the fluid becomes smaller and vice-versa.

Fig. 5.21: Steady flow of fluid through a conduit of a different area of cross-sections.

5.5.5 Bernoulli's Theorem and Its Applications

Bernoulli's theorem was derived by Swiss mathematician Daniel Bernoulli in 1738. It is a principle of conservation of energy and it states that the total mechanical energy of the non-viscous and incompressible fluid flowing in a streamlined motion from one point to another remains constant at every point of its path throughout its flow i.e.,

$$
P\frac{m}{\rho} + mgh + \frac{1}{2}mV^2 = \text{Constant}
$$

$$
\frac{P}{\rho} + gh + \frac{1}{2}V^2 = \text{Constant}
$$

Or

$$
P + \rho gh + \frac{1}{2}\rho V^2 = \text{Constant}
$$

W = ρ
W = $\frac{Mass \text{ of the fluid}}{\text{Volume of the fluid}}$

Here, the total mechanical energy is the sum of energy associated with the fluid pressure (or Pressure

energy, W = pressure \times Δ (volume of the fluid) = pressure \times $\Delta\left(\frac{\text{Mass of the fluid}}{\text{Density of the fluid}}\right)$), gravitational potential energy due to the elevation of a conduit from the surface of the earth ($PE = mgh$), and kinetic energy of the fluid motion (KE = $\frac{1}{2}$ mV²).

Fig. 5.22: Flow of fluid through an elevated conduit explained using Bernoulli's theorem
Bernoulli's theorem is strictly valid for non-viscous fluids or fluid with zero viscosity that flows in streamlines. It does not apply to turbulent flow. Note that, if the conduit or tube is placed horizontally on the surface of the earth, $h_1 = h_2 = h = 0$, and then the above equation becomes –

 $\frac{P}{\rho g} + \frac{1}{2} \frac{V^2}{g}$ = Constant

Applications of Bernoulli's Theorem

Bernoulli's theorem has several applications. It is used in the Venturimeter (or flowmeter), an aerofoil lift, the Bunsen burner, in working of an air-plane and the lift. Venturimeter is a device that is used for measuring the flow rate of the fluid through conduits or pipes. Bernoulli's theorem is also used in designing a carburetor of automobiles, filter pumps, atomizers, and sprayers. The action of a paintgun, perfume spray, or insect sprayer is based on this theorem. Bernoulli's principle also explains the Magnus effect that plays an important role in golf, cricket, soccer, tennis, etc. The spinning ball used in these sports curves due to the Magnus effect. This principle also explains why roofs are blown off during tornados or hurricanes or how the ships sail through water. When a fast-moving train passes it pulls nearby objects or when two nearby row boats move parallel to each other, they are pulled towards each other. Bernoulli's principle explains both of these. The phenomenon of blood flow in the vessels and heart attack due to atherosclerosis can be explained with the help of Bernoulli's theorem.

Case-Study (Environmental/Sustainability/Social/Ethical Issues)

The dynamic lift due to the spinning of a body is known as the Magnus effect. Due to this effect, there is a variation in the trajectory of the rotating object in the moving fluid and the rough surface of the object drags more air as compared to a smooth surface. When the ball with a rough surface is thrown with a spin, it drags more air with it. The rough surface allows acceleration of the air molecules which are above the ball and deceleration of the air molecules which are below the ball. Due to this, there are fewer air molecules above the ball and more air molecules below the ball creating a lift on the ball and the ball covers more distance. The golf ball has more dimples on its surface to create the Magnus effect.

Create Inquisitiveness and Curiosity

Does the shape of a tube or a duct change the value of Reynolds numbers? Take an example of a circular tube, a square duct, and a rectangular duct and figure out which type of tube or duct will be better for a liquid flow and a gas flow.

Solved Problems

Problem-1: Consider that water is flowing in a fire hose with a velocity of 2.0 m/s and pressure of 250000 Pa. The pressure decreases to atmospheric pressure (101300 Pa) at the nozzle. Assume that there is no change in height, calculate the velocity of the water leaving the nozzle. The density of water is 1000 kg.m⁻³. Gravitational acceleration 'g' = 9.8 m s⁻².

Solution:

According to Bernoulli's principle,

$$
P_1 + \rho g h_1 + \frac{1}{2} \rho V_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho V_2^2
$$

Since, $h_1 = h_2$, we can rewrite this equation as -

$$
P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2
$$

\n
$$
\therefore V_2 = \sqrt{\frac{2}{\rho} \left(\frac{1}{2}\rho V_1^2 + P_1 - P_2\right)} = \sqrt{\frac{2}{(10^3)} \left(\frac{1}{2} \times (10^3) \times 2^2\right) + 250000 - 101300}
$$

\n
$$
= \sqrt{301.4} = 17.36 \text{ ms}^{-1}
$$

UNIT SUMMARY

- Restoring force per unit area produced in a body due to deformation is called stress and the fractional change in configuration of a body is called strain.
- Hooke's law: States that within an elastic limit, stress is proportional to strain.
- Young's modulus $Y = MgL/\pi r^2 l$, Bulk modulus $B = -V\Delta P/\Delta V$ and modulus of rigidity $\eta = FL/Ax$
- Force per unit area exerted normally on the fluid is called fluid pressure. Atmospheric pressure is the pressure exerted by the air on the earth surface that is the force applied by air per unit area on the earth. 1 atmospheric pressure = 1.013×10^5 pascal. Gauge pressure is excess pressure than atmospheric pressure.
- Fortin's barometer is used to measure atmospheric pressure.
- Surface tension is a property of a liquid surface due to which it behaves as the stretched membrane. Mathematically surface tension is defined as force per unit length on a line that imagines on a liquid surface and perpendicular to this line, along the liquid surface.
- The intermolecular force of attraction between molecules of the same substance is called cohesive force and adhesive force is the force between molecules of different substances.
- The angle of contact is the angle between tangent at solid and tangent at the liquid surface at the contact.

$$
2S\cos\theta
$$

- The rise of liquid in the capillary is given as $h = \frac{2S\cos\theta}{r\rho g}$ \bullet
- The surface tension of liquid decreases with an increase in temperature.
- Viscosity is a measure of internal shear resistance of fluids to flow and it is temperaturedependent property of fluids.
- Viscous force is a drag force within the fluid, which opposes the relative motion of one fluid layer over another.
- Reynolds number is the ratio of the inertial forces to the viscous forces of the fluid.
- \bullet In laminar flow, the values of velocity, pressure, and other quantities at a point in space do not fluctuate randomly with time.

Bernoulli's principle is derived from the conservation of energy for fluids. It states that the \bullet total mechanical energy per unit mass of a non-viscous and incompressible fluid flow is constant.

EXERCISES

Objective Questions (A)

(A) Answers of Objective Questions

- A5.2 $\Delta L = MgL/\pi r^2 Y$
- $A5.3 \quad \Delta V/V = \Delta P/B$
- $A5.7$ poise and stokes
- Bernoulli's principle A5.8

A5.9 Energy

A5.10 Stokes' law

A5.11 48 (\cdot 1 poise = 0.1 N-s.m⁻²)

Subjective Questions (B)

- A steel wire of length 2.5m is stretched through 1.0mm. The cross-section area of the wire is 5.1 5.0mm². Find (1) the stress (2) weight or load in the wire. Young's modulus of steel $Y = 2 \times 10^{11}$ $N/m²$. [LOD2]
- 5.2 Find (1) change in pressure (2) and final pressure on an air-filled balloon from the following data. Initial volume = 8×10^{-3} m³, initial pressure = 10^{5} N/m, decrease in volume = 10^{-3} m³, the compressibility of air = $7.65 \times 10^{-6} \text{N}^{-1} \text{m}^2$. [LOD3]
- A copper wire of length 2m and cross-sectional area 0.02cm² is clamped at one end. A load of 5.3 2kg is attached to the free end. Find elongation of the wire. Young's modulus of copper $=$ 1×10^{11} N/m², g = 10m/s². $[LOD1]$
- 5.4 Find (1) the absolute pressure (2) and gauge pressure at the bottom of a tank filled with water up to a height of 10m. Water surface at top of the tank is open to the atmosphere. [LOD2]
- 5.5 A capillary of glass is dipped into water and water rises 10cm in the capillary. Determine the radius of the capillary. (Surface tension of water S= 7×10^{-2} N/m, angle of contact = 0°, g = 10m/s²) $[LOD1]$
- If the height of water rise in a glass capillary of sufficient length is doubled than the first capillary 5.6 of the same material. Then the ratio of radii (r_1/r_2) is _____. $[LOD2]$
- Give the difference between the laminar and turbulent fluid flow. 5.7 $[LOD1]$
- 5.8 The terminal velocity of a copper ball of radius 2.5 mm falling in a container of glycerine at 25 $\rm{^{\circ}C}$ is 5.6 cm s^{-1} . Compute the viscosity of the glycerine at 25 °C. The density of glycerine is 1260 kg m⁻³, the density of copper is 8.9×10^3 kg m⁻³. [LOD1]
- 5.9 Why does the fluid viscosity depend on temperature?
- 5.10 Discuss the importance of Reynolds number.
- A fuel ethanol is flowing in a pipe at a velocity of 100 cm.s⁻¹ at atmospheric pressure through a 5.11 refinery. The refinery needs the ethanol to be at a pressure of 2 atm (202600 Pa) on a lower level. Calculate how much the pipe has to drop in height to achieve this pressure. Assume that the velocity does not change. The density of ethanol is 789 kg.m⁻³ and the value of 'g' is 9.8 m s⁻².

 $[LOD2]$

 $[LOD2]$

 $[LOD1]$

Answers of Subjective Questions (B)

A5.1 Stress = Y Strain =
$$
Y \frac{\Delta L}{L}
$$
 = 8 × 10⁷N/m² and F = A (Stress) = 400N

A5.2 $\Delta P = \frac{\Delta V}{VK} = 0.16$ atm and $P_1 = P_2 + \Delta P = 1.15$ atm A5.3 $\Delta L = MgL/AY = 0.2mm$ A5.4 $P = P_{atm} + \rho gh = 2atm$ and $P - P_{atm} = \rho gh = 1atm$ A5.5 $r = 2S/h \rho g = 0.14$ mm A5.6 $h_1r_1 = h_2r_2$ and $r_1 : r_2 = 2 : 1$ A5.8 1.8569 Pa-s A5.11 -13.1 m (or 13.1 m lower) (Hint: Use the Bernoulli equation.)

PRACTICALS

2_{-} To determine force constant of a spring using Hooke's Law.

Practical significance

The stiffness of the spring is determined by its force constant or spring constant. Higher spring constant means a stiffer spring that is harder to stretch and it requires larger force for stretching it. Force constant of a spring is a characteristic feature of a spring due to its elastic properties. This can be determined using Hooke's Law. Effective mass of the spring can also be determined using this experiment.

Relevant theory

For theory, refer to Unit 5 (Section: 5.1.2)

Formula: Spring constant, $K = \frac{4\pi^2(m_1 - m_2)}{\left(T_1^2 - T_2^2\right)}$

Where T_i = Time period of oscillations of a helical spring for load 1;

 $T₂$ = Time period of oscillations of a helical spring for load 2;

 m_1 = mass of the load 1;

 m_2 = mass of the load 2;

 $K =$ force constant or spring constant = $\frac{\text{Restoring force}}{\text{extension}}$

Practical outcomes (PrO)

The practical outcomes are derived from the curriculum of this course:

- PrO1: Acquire skills in setting up the experiment and precise measurement of periodic time of oscillations of a helical spring using stop watch.
- PrO2: Describe and verify Hooke's law for elasticity in solids and determine the force constant of a helical spring.
- PrO3: Operate and control the required equipment with proper precautions either in a group or individually.

Practical setup (drawing/sketch/circuit diagram/work situation)

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Fig. 5.23: Experimental setup for determining the force constant of a spring using Hooke's law

Resources required

Precautions

The number of oscillations, n, should be chosen such that the oscillating mass takes more than 1. 10.0 sec to complete them, if the stop watch has least count of 0.1 sec. This will ensure that the percentage error in the measurement of time is below 1%.

- The equations to calculate the force constant here, remain true for small amplitude of $2.$ oscillations or small extension of the spring within the elastic limits (Hooke's law). So, take special care while releasing the load from its rest position to either side, upward or downward, very gently.
- The effect of buoyancy of air and viscous drag due to this can affect the damping of the $3.$ oscillations of the spring. These effects can be reduced by taking a small and slightly stiffer spring of high-density material.

Suggested procedure

- Suspend the helical spring PQ from the rigid support such that the pointer is at its free end Q $1.$ as shown in figure.
- $2.$ Now, fix the measuring scale vertically such that it remains close to the spring and the pointer attached to the spring can move freely over the scale without touching it. Record the least count of the measuring scale which is usually 1mm or 0.1 cm.
- $3.$ Similarly, record the least count of the stop watch.
- $\overline{4}$. Now, gently suspend the load or slotted weight with mass (m) on the hanger. Wait till the pointer stops moving for a given load 'm₁'. This is its equilibrium position for the mass m₁.
- Pull the load 'm,' slightly downwards and then release it gently so that it starts oscillations in a 5. vertical plane about its equilibrium position 's'.
- Start the stop-watch as the pointer just crosses its mean position (this is the equilibrium position) 6. and simultaneously begin to count the time for 'n' oscillations. Note the time (t) for n oscillations of the oscillating load 'm.'.
- 7. Repeat the observation at least thrice for the same load and note the time for 'n' oscillations. Find the mean time (t_m) , for n oscillations of a helical spring and calculate the time for one oscillation, i.e., the periodic time, $T (= t_m/n)$ of oscillating helical spring with a load m.
- Repeat steps 5 to 7 for two more slotted weights and calculate the periodic time of oscillations 8. for each weight and record them in the observation table.
- 9. Calculate the value of spring constant (K_1, K_2, K_3) for each load and find out the mean value of spring constant K of the given helical spring.
- 10. Alternately, the value of K can also be determined by plotting a graph of T^2 vs. m with 'T²' on y-axis and 'm' on x-axis, which will be a straight line with an intercept and slope. The value of spring constant can be determined from the slope of a straight line and effective mass of the helical spring can be determined using the intercept.

Observations and calculations

Observations

- $1.$ Least count of the measuring scale = μ mm = μ cm
- $2.$ Least count of the stop-watch $=$ ____ sec
- $3₁$ Mass of load 1, m₁ = $___$ g = $___$ kg
- Mass of load 2, $m_2 =$ ______ g = _____ kg 4.
- Mass of load 3, m₃ = $__ g = __ g = __ kg$ 5.

Observation table

Calculation

From theory: (calculate force constants for load 'm' and then take their average)

$$
K_1 = \frac{4\pi^2 (m_1 - m_2)}{(T_1^2 - T_2^2)}; K_2 = \frac{4\pi^2 (m_2 - m_3)}{(T_2^2 - T_3^2)}; K_3 = \frac{4\pi^2 (m_1 - m_3)}{(T_1^2 - T_3^2)}
$$

And

$$
K = \frac{K_1 + K_2 + K_3}{3}
$$

$$
= \frac{N/m}{N}
$$

From graph: $K = \frac{4\pi^2}{\text{slope}}$
Error in the value of K from graph between T² (on Y-axis) vs. m (on X-axis):
From *m* the value of K from graph between T² (on Y-axis) vs. m (on X-axis):

Error in the value of
$$
K = \frac{dN}{K} = \frac{dN}{d}
$$
 where M/m is the number of $K = \frac{dN}{d}$.

Results and/or interpretation

- $1.$ The force constant of a helical spring calculated by using Hooke's law, __N/m.
- $2.$ Error in the value of K from the graph $=$ ______ %
- Effective mass of the helical spring, $m_0 =$ _______ kg $3.$

Conclusions and/or validation (To be filled by student)

Practical related questions (Use separate sheet for answer)

- 1. What is the significance of a force constant 11.2 N/m?
- $2.$ How does the force constant of springs change when they are attached with each other in series?
- $\overline{3}$. What is the force constant of a system of three springs attached in parallel with each other?
- $4.$ What are the limitations of Hooke's law?
- 5. List out the real-life applications in which Hooke's law is applied.

Disposal of waste

Classify the waste materials to be thrown in this experiment in the following bins:

Environment friendly approach: reuse, reduce and recycle

Apparatus for this practical are reusables for sufficiently long time if they are operated carefully.

Suggested assessment scheme (To be filled by teacher)

The given performance indicators should serve as a guideline for assessment regarding process and product related marks.

* Marks and percentage weightages for product and process assessment will be decided by the teacher.

To find the viscosity of a given liquid (Glycerine) by Stokes' law. $2.$

Practical significance

Stokes' law relates the drag forces experienced by the falling spherical object in the fluid to the velocity of the sphere in the fluid of known viscosity. With this experiment, terminal velocity of the small and spherical object can be determined provided viscosity of the fluid is known. Measuring terminal

velocity and diameter of a spherical object, we can find coefficient of viscosity or viscosity of an unknown fluid.

Relevant theory

For theory, refer to Unit 5 (Section: 5.4.2)

Formula:
$$
V_t = \frac{2 r^2 g}{9 \eta} (\rho - \rho_f) \therefore \eta = \frac{2 r^2 g}{9 V_t} g(\rho - \rho_f)
$$
 (From theory)
and $\eta = \frac{2}{3} \frac{1}{\text{slope}} g(\rho - \rho_f)$ (From graph between V_t vs. r^2)

Where $r =$ radius of the spherical object;

 $V₊$ = terminal velocity of the spherical object;

 $g =$ gravitational acceleration;

 η = coefficient of viscosity or viscosity of the fluid;

 ρ = density of the spherical object;

 ρ_f = density of the fluid

Practical outcomes (PrO)

The practical outcomes are derived from the curriculum of this course:

- PrO1: Acquire skills in adjusting and marking the positions of rubber bands, in accurate use of vernier calipers and stop watch to find the diameter of the spherical object and time it takes for free-fall.
- PrO2: Describe Stokes' law for viscous fluids and determine viscosity of a given fluid in allotted time.
- PrO3: Operate and control the required equipment with proper precautions either in a group or individually.

Practical setup (drawing/sketch/circuit diagram/work situation)

Fig. 5.24: Experimental setup for measuring the viscosity of a given liquid using Stokes' law

Resources required

Precautions

- The radius of the wide bore glass tube which contains viscous fluid should be much larger than 1. the radius of the spherical balls.
- $2.$ The spherical balls should not touch the sides of the glass tube while falling freely in the viscous fluid. Use of forceps is recommended to drop the ball in the tube.
- The spherical ball should be dropped gently in the glass tube filled with viscous fluid. They $3.$ should be rinsed with the experimental viscous fluid (glycerine) in a watch glass to avoid forming air bubbles as they enter the fluid column.
- The viscous fluid should be poured carefully in the fluid column. Follow precautionary guide-4. lines when dealing with any chemicals, be it toxic or non-toxic.

Suggested procedure

- 1. Set up the apparatus as shown in the diagram at room temperature and pressure. Ensure that there are no air bubbles inside the viscous fluid in the glass tube. Measure the room temperature using thermometer.
- Place three rubber bands at A, B and C around the wide bore glass tube such that the glass tube 2. is divided into four portions. Keep the distance $AB = BC$. The rubber band at A should be measured from the mouth of the wide bore glass tube.
- $3.$ Use vernier caliper to measure the diameter of given spherical objects (steel balls).
- $4.$ Fix a short inlet tube or funnel vertically at the open end of the wide bore glass tube so that steel balls fall freely without touching the sides of the glass tube. Now, drop a clean and dry spherical object (or steel balls) of different radii gently into the fluid. You may need a set of four or five identical steel balls of same radii (r), so make separate sets of steel balls of same radii.
- 5. Use two stop watches and start both of them simultaneously as soon as the steel ball passes through the rubber band at position at A. When the ball passes through the band at B, stop one of the watches. Stop the second watch when the ball crosses the band at C.
- 6. Note the time the steel ball takes to fall from A to B and A to C as 't₁' and 't₂', respectively. If the steel ball achieves terminal velocity, then $t₂ = 2 t₁$. If the steel ball does not achieve terminal velocity, then repeat the experiment with the steel ball of same radius while adjusting the positions of rubber bands.
- 7. Repeat the experiment for other steel balls of different radii (r_1) .
- Measure the length AB and AC if the rubber bands are adjusted for achieving terminal velocity. 8.
- 9. Determine the terminal velocity (V_+) for each steel ball of different radii.
- 10. Plot the graph between terminal velocity (V_+) on Y-axis and square of the radius of the spherical object, r^2 on X-axis. Find the slope of a straight line and hence determine the coefficient of viscosity of the fluid using the relation given above.

Observations and calculations

Observations

- $1.$ Temperature of the fluid $(T) =$ $\frac{C}{T}$.
- Density of material of steel balls (ρ) = ___ kg-m⁻³ $2.$
- Density of the viscous fluid used in the tube $(\rho_e) =$ ____ kg-m⁻³ $3.$
- $\overline{4}$. Internal diameter of the wide bore glass tube or column = $___$ cm = $___$ m
- Length of the glass tube = $\text{cm} = \text{cm} = \text{cm}$ $5.$
- Distance between A and B = μ cm = μ m 6.
- Distance between B and C = $\text{cm} = \text{cm} = \text{cm}$ $7.$
- Average distance (h) between two consecutive rubber bands = $\text{cm} = \text{cm} = \text{cm}$ 8.
- Acceleration due to gravity at the place of experiment (g) = ___ cm-s⁻² = ___ m-s⁻² 9.
- 10. Least count of stop-watch = $___\$ s

Observation table

Calculation

From theory: (calculate for each spherical ball of different diameter and then take average of coefficient of viscosity)

(From theory)

$$
\eta = \frac{2 r^2}{9 V_t} g(\rho - \rho_f) \text{ (From theory)}= \underline{\qquad} \text{Ns-m}^{-2}
$$

From graph between V_1 (on Y-axis) vs. r^2 (on X-axis):

$$
\eta = \frac{2}{9} \frac{1}{\text{slope}} g(\rho - \rho_f) = \underline{\qquad} \text{Ns-m}^{-2}
$$

Results and/or interpretation

The coefficient of viscosity of the given viscous fluid (Glycerine) calculated by Stokes' law at temperature, $C =$ Ns-m⁻².

Conclusions and/or validation (To be filled by student)

Practical related questions (Use separate sheet for answer)

- Why do you need a spherical object for this experiment? 1.
- $2.$ When does the free-falling spherical object achieve terminal velocity?
- $3₁$ What are the limitations of Stokes' law?
- $4.$ How does the value of viscosity of glycerine change with temperature?
- What will happen if the density of spherical object is less than the density of the fluid? 5.

Disposal of waste

Classify the waste materials to be thrown in this experiment in the following bins:

Environment friendly approach: reuse, reduce and recycle

Use a sufficiently strong magnet to retrieve steel balls back from fluid column for their reuse. Unless required, the viscous fluid once poured in fluid column can be reused for multiple times. For different fluids, few similar glass tubes can be purchased and reused.

Suggested assessment scheme (To be filled by teacher)

The given performance indicators should serve as a guideline for assessment regarding process and product related marks.

* Marks and percentage weightages for product and process assessment will be decided by the teacher.

3. To determine atmospheric pressure at a plane using Fortin's barometer.

Practical significance

Students can use this barometer in the measurement of atmospheric pressure in different workshops, laboratories and engineering jobs in future.

Relevant theory

As atmospheric pressure changes the mercury level in the cistern of Fortin's barometer changes accordingly, When the pressure increases, it causes the mercury level to fall Fortin's barometer allows the level of mercury in the cistern to always be kept at zero on the main scale while taking the reading. The leather bag is filled with mercury and the mercury level is raised or dropped with the help of a screw jack. After made this adjustment, the top of the mercury column in the glass tube is then measured by bringing the lower edge of the vernier so that it just touches the top of the mercury column or meniscus of mercury. Reading is then taken by measuring the main scale reading and coincide the division of the vernier scale with the main scale. So, the height of the mercury column is equal to the sum of the main scale and vernier scale reading

Or total reading $h = \text{main scale reading} + \text{coincide, division of vernier} \times \text{Least count (L.C.),}$

Thus, atmospheric pressure $P_{atm} = \rho gh$

If the density of mercury ρ and g is constant then $P_{atm} \propto h$

Least count (L.C.) of vernier

 $=$ one division of the main scale divided by the total number of divisions on a vernier scale

L.C. =
$$
\frac{\text{lmm}}{10} = 0.1 \text{mm}
$$

Practical outcomes (PrO)

- PrO1: Apply the principle of Fortin's barometer
- PrO2: Find atmospheric pressure using Fortin's barometer
- PrO3: Measure atmospheric pressure at any place on the earth surface in life

Practical setup (drawing/sketch/circuit diagram/work situation)

Refer the schematic diagram of Fortin's barometer in Section 5.2.4

Resources required

Precautions

- $1.$ While adjusting the mercury level to point zero, the adjusting screw should not be turned too quickly.
- $2.$ There should no variation in the temperature of the room in which pressure is measured.
- There should not be exposed to sunlight otherwise it will be difficult to judge mercury level in $3.$ the brass tube.
- It should be tested to ensure that no air has entered the vacuum space at the top of the mercury $4.$ column.

Suggested procedure

- $1.$ Lightly tap the barometer so that the height of the meniscus (shape of the top surface of the mercury column in the glass tube) is neither too large nor too small.
- $2.$ Use the zero adjusting knob so that the top of the mercury reservoir just touches the tip of the zeroing peg or tip of the ivory pointer and zero of the main scale.
- Adjust the height of the movable scale or vernier scale so that the bottom of the front and back $3.$ of the scale is just even with the top of the meniscus.
- Read the height of the mercury column in the glass tube using the vernier scale and reading of $4.$ the main scale. Reading is noted on the main scale and coincide, division of vernier scale, then total reading will give the height of mercury column (h).
- 5. Repeat step 4 three times and then take mean of readings of height (h).
- $6.$ Read the temperature using the thermometer mounted on the barometer.

Observations and calculations

Observation table for height (h) of mercury column

Calculations

Mean reading of height h = mm Atmospheric pressure is calculated by the formula given as $P_{atm} = \rho gh$

Results and/or interpretation

Atmospheric pressure measured in laboratory $P_{\text{atm}} =$ pascal

Conclusions and/or validation

(To be filled by student)

Practical related questions or viva-voce questions

- Define atmospheric pressure. $\mathbf{1}$
- $\overline{2}$ Explain variation in atmospheric pressure with altitude.
- \mathfrak{Z} Explain use of screw at bottom of the cistern.
- $\overline{4}$ Discuss atmospheric pressure variation with acceleration due to gravity 'g'.

Suggested assessment scheme

(To be filled by teacher)

The given performance indicators should serve as a guideline for assessment regarding process and product-related marks.

* Marks and percentage weightage for product and process assessment will be decided by the teacher.

KNOW MORE

 $1.$ Poisson's ratio: The ratio of fractional change in diameter (or lateral strain) to the fractional change in length (or longitudinal strain) of wire is called Poisson's ratio.

a.
$$
\sigma = \frac{\text{change in diameter/original diameter}}{\text{change in length/original length}} = \frac{\Delta D}{\Delta L_f} = \frac{L \Delta D}{\text{D} \Delta L}
$$

Theoretically, the limiting value of σ is between -1 and 0.5 \mathbf{b} .

- $2.$ Pascal's law: states that pressure is applied to any point in an enclosed fluid at rest that pressure transmitted equally at each point of the fluid.
- Relative density or specific gravity is the ratio of 3. the density of a substance to the density of water at 4° C.
- $\overline{4}$. Ascent formula: Let a capillary tube of radius r is dipped in a liquid of surface tension S and the liquid rises by height h, as shown in (Fig. 5.25) then,

Weight of liquid rise in capillary $=$ force due to surface tension or Mg = $(S \cos \theta)2\pi r$

Here mass of liquid M = density \times volume = $\rho \times \pi r^2 h$

So, $\rho \times \pi r^2$ hg = (S cos θ) $2\pi r$ or h = (2S cos θ)/ ρr g (Here ρ = density of liquid)

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Thixotropic fluids are those which have time-dependent shear thinning property, i.e., a viscosity 5. of these non-Newtonian fluids decreases under shear strain. Thixotropic fluids are usually static viscous fluids under normal conditions, but when agitated or shaken or shear-stressed, their viscosity reduces. Some clays, cytoplasm, drilling muds are natural examples of thixotropic fluids. Thixotropic ink is a key feature of a pen used for writing during zero-gravity space flights. For electronic manufacturing printing processes, thixotropic solder pastes are used. Manuka Honey is an example of thixotropic fluid. Paraffin oil, cream, paints, ketchup, toothpaste, etc. are also examples of thixotropic fluids. It is important to note that pseudoplastic behaviour in the fluid is synonymous with thixotropic behaviour except the time-dependence of shear thinning.

Innovative Practical/Projects/Activities

- $\mathbf{1}$. To find the atmospheric pressure by Torricelli barometer.
- To find the viscosity coefficient of honey. $\overline{2}$.
- $\overline{3}$. To find gauge pressure of a gas by the manometer.
- Prepare a PowerPoint presentation about different methods to measure the dynamic and kinematic $\overline{4}$. viscosity of various fluids like water, castor oil, organic liquids, paint, polymers, crude oil, etc.
- 5. Discuss among the group of students about Magnus effect on sports balls and present the work in the form of a PowerPoint presentation.

REFERENCES AND SUGGESTED READINGS

References

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- $\overline{3}$. M. E. Browne, Schaum's Outline of Physics For Engineering And Science, 4th Edition, McGraw-Hill, 2019.
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- 10. Swayam Prabha, INFLIBNET, India, $https://$ www.swayamprabha.gov.in/index.php/

Suggested Learning Resources for Practical

- $\mathbf{1}$. C. L. Arora, B.Sc. Practical Physics, S. Chand Limited, 2001.
- $2.$ G. L. Squires, Practical Physics, Cambridge University Press, 2001.

For virtual lab on viscosity

6 Heat and Thermometry

UNIT SPECIFICS

This unit focusses on the following aspects of basic physics:

- Concepts of heat and temperature
- Modes of heat transfer with examples and specific heat
- Scales of temperature and their relationship, types of thermometers and their uses
- Expansion of solids, liquids, and gases, coefficient of linear, surface, and cubical expansions and the relation amongst them,
- Concepts of thermal conductivity and its applications

RATIONALE

Heat is thermal energy and temperature is the quantification of this energy. Here, you will understand the concepts of these quantities and be able to differentiate three modes of heat transfer using some examples. You will study the inherent properties like the specific heat of materials and be able to differentiate different temperature scales and their relationship. You will also understand different types of thermometers Finally, you will study the temperature effects on gas, liquid and solid materials in terms of their expansion, thermal conductivity. You will be able to realize the various engineering applications.

PRE-REQUISITES

Physics – High School level Physics Mathematics – Basic Algebra Other – Basic knowledge of Computer

UNIT OUTCOMES

- U6-O1: Identify heat, temperature, thermal conductivity, heat capacity with its appropriate unit and dimensions.
- U6-O2: Differentiate the modes of heat transfer and temperature scales. Convert temperatures of substances from one scale to another scale
- U6-O3: Apply the concept of thermal conduction to find a relation between linear, surface, and volume expansion in solids.
- U6-O4: Describe the importance of thermal conductivity in engineering applications and identify good and poor conductors of heat.

Mapping of unit outcomes with the course outcomes:

61 **HEAT TRANSFER AND TEMPERATURE MEASUREMENT**

Interesting Facts

Do you know that most varieties of soft cheese have above 40% water content it and that is the reason why cheese on pizza takes longer to heat up and cool down? It is due to the heat capacity of water present in cheese that scalds your palette if you eat the hot-served cheese part of a pizza. The crust of the pizza, however, heats up and cools down relatively faster, and hence comparatively, you don't burn your mouth from the crust.

$6.1.1$ **Concepts of Heat and Temperature**

Heat and temperature are not the same quantities, although we sometimes use them interchangeably. Heat is an overall potential and kinetic energy of molecular motions and is measured in joule or calorie, whereas temperature is a measure of the average kinetic energy of molecular motions and it is a measure of heat energy. SI unit of temperature is kelvin, its other units are degree Celsius (°C) and degree Fahrenheit (°F). The temperature of an object doesn't depend on the type or number of molecules; it only measures the speed of atoms or molecules in an object as the intensity of heat energy. The symbol for heat energy is 'Q' and that for temperature is 'T'. The content of heat in an object makes it either hot or cold. Heat energy always flows from hotter or warmer substance or region to cooler substance or region i.e., it flows from a region or substance of higher temperature to that of lower temperature. Heat energy is the ability to perform work while the temperature is a measure of the degree of heat (i.e., hotness or coldness of an object).

Modes of Heat Transfer: Conduction, Convection and Radiation $6.1.2$

Heat transfers in three different ways: (a) conduction (b) convection (c) radiation.

(a) Conduction: In solids like metals, heat transfers via molecular vibrations of the molecules which are in direct contact with each other. In this case, heat is transferred without the actual movement of particles. This method of heat transfer is known as conduction. The medium is required for heat transfer in case of conduction in solids.

Substances that conduct heat are known as thermal conductors.

(b) Convection: In fluids like water or air, the transfer of heat takes place due to the actual movement of particles from one place to another is convection. This is known as convection. The heat transfer through convection cannot occur in a vacuum.

In both conduction and convection, heat spreads across all parts of the medium while trying to reach an equilibrium state.

(c) Radiation: Radiation can occur in a medium or vacuum. When heat transfers through radiation, the space in between is not heated up. This means that the spreading of heat via radiation in the form of electromagnetic waves is targeted and localized.

Photo credits : This Photo is licensed under CC BY-SA Fig. 6.1: Modes of heat transfer: (a) conduction (b) convection (c) radiation

Let's consider this example in which we boil water in a steel pan on a hotplate or gas stove. Since the bottom of the pan is placed on the hotplate or the gas stove, heat energy transfers from there to the bottom of the pan through (thermal) radiation at first. The heat eventually spreads to the middle, top, and handle of the pan through conduction where the molecules in the pan do not move from their positions. Heat transfer takes place between molecules in direct contact with each other by molecular vibrations. The water in the pan then starts heating up at the bottom due to the high temperature there. Since molecules in the water are not bound to each other as rigidly as in the steel pan, they become more mobile due to gained energy. They carry heat energy with them to the cooler region. The colder region is denser than the warmer region in water, molecules from the cold region move downwards to the bottom of the pan and lighter molecules from the hot region move upwards to the colder part. This movement of molecules from cold to the warm region continues until equilibrium is reached. This process of heating the water is convection which involves the actual movement of water molecules. Instead of using water, we can consider oil, too.

Water in waterbodies gets heated up under direct sunlight which is due to radiation and convection. Due to convection and radiation, we can explain why the desert area is warmer in the day and colder during the night. Metal spoon kept in a glass of hot milk eventually becomes warm, too because of conduction. To avoid getting burns due to conduction in hot utensils during cooking, they are equipped with thermally insulated handles.

$6.1.3$ **Specific Heats**

Heat capacity: It is the amount of heat required to raise (or reduce) the temperature of a substance by 1 K. The symbol for heat capacity is \mathbb{C}^{\prime} and its SI unit is J.K⁻¹.

Heat supplied to (or taken out of) a substance, $Q = C\Delta T = C (T_2 - T_1)$; where, $\Delta T =$ change in temperature, T_2 = final temperature and T_1 = initial temperature.

Specific heat or Specific heat capacity: It is the amount of heat required to raise the temperature of 1 gram of a substance by 1 °C. The unit of specific heat is 1 calg⁻¹ °C⁻¹ or 1 Jg⁻¹ °C⁻¹. OR it is the amount of heat required to raise the temperature of 1 kilogram of a substance by 1 kelvin. SI unit of specific heat is J kg⁻¹ K⁻¹. In the FPS unit system, the unit of specific heat is BTU lb⁻¹ oF⁻¹ (British thermal unit per pound per Fahrenheit)

In general, specific heat is the amount of heat supplied to (or taken out of) a unit mass of a substance to increase (or decrease) its temperature by one degree in the thermodynamic process.

Heat supplied to (or taken out of) a substance, Q = m C ΔT = m C (T₂ - T₁); where, m = mass of a substance, ΔT = change in temperature, T_2 = final temperature and T_1 = initial temperature.

The specific heat of water at room temperature and atmospheric pressure is calg⁻¹ \circ C⁻¹ or 4.186 Jg⁻¹ °C⁻¹.

There are two values of specific heat or specific heat capacity:

- (1) Specific heat at constant pressure (C_p) : It is the amount of heat required to raise a temperature of a unit mass by a unit degree on a thermodynamic scale at constant pressure.
- (2) Specific heat at constant volume (C_v) : It is the amount of heat required to raise a temperature of a unit mass by a unit degree on a thermodynamic scale at constant volume.

Molar heat capacity: It is the amount of heat required to raise the temperature of 1 mole of a substance by 1 Kelvin. Its SI unit is J mol⁻¹ K⁻¹. It is denoted by C_m .

Heat supplied, Q = n C_m ΔT = n C_m (T₂ - T₁); where, n = number of moles of a substance, ΔT = change in temperature, T_2 = final temperature and T_1 = initial temperature.

The molar specific heat of a substance is an inherent characteristic of a substance that is independent of the shape and size of the substance. The value of the molar specific heat of a substance is generally smaller than the specific heat of the substance. For example, the specific heat of paraffin wax is about 2500 J kg⁻¹ K⁻¹ but its molar heat capacity is about 600 J mol⁻¹ K⁻¹.

Relation between C_p and C_v :

1. The ratio of the specific heats, $\gamma = \frac{C_p}{C_w}$. This ratio of specific heats is also known as isentropic expansion factor or adiabatic index.

2. $C_p - C_v = R$, where R = universal gas constant. Note here that, C_p is always greater than C_v (i.e., $C_p > C_V$).

Scales of Temperatures and Their Relationship $6.1.4$

Four temperature scales are still in use: (a) Celsius (or Centigrade) scale (b) Fahrenheit scale (c) Kelvin scale (d) Rankine scale.

(a) Celsius (or Centigrade) scale: In the Celsius scale, the freezing temperature of the water is 0° C and boiling temperature is 100 $^{\circ}$ C. (b) Fahrenheit scale: In the Fahrenheit scale, water freezes at a temperature of 32 \textdegree F and boils at 212 \textdegree F. (c) Kelvin scale: In Kelvin scale, the freezing point of water is 273.15 K and the boiling point is 373.15 K. Here, the zero is set to absolute zero (0 K) i.e., at -273.15 °C or -459.67 °F. Note that $(°)$ symbol to show the degree of temperature is not used in Kelvin scale, e.g., it is 100 K and NOT 100 °K. (d) Rankine scale: In Rankine scale, zero is set to absolute zero and water freezes at 491.67 $\,^{\circ}$ R and boils at 671.67 $\,^{\circ}$ R.

Conversion between Fahrenheit scale and Celsius scale

$$
^{\circ}F = \frac{9}{5} \,^{\circ}C + 32
$$

Note here that, - 40 Fahrenheit scale equals Celsius scale i.e., - $40^{\circ}F = -40^{\circ}C$

Conversion between Kelvin scale and Celsius scale

$$
K=273.15+{\circ}C
$$

Conversion between Rankine scale and Fahrenheit scale

$$
{}^{\circ}\mathrm{R} = {}^{\circ}\mathrm{F} + 459.67
$$

Conversion between Rankine scale and Kelvin scale

Fig. 6.2: Temperature scales

Types of Thermometers and Their Uses 6.1.5

The thermometer is a device or tool to measure the temperature of a body or an object and is kept in direct contact with the object or in the vicinity of the object. They are of two types: (a) electrical (b) non-electrical. Mercury thermometers and bimetal thermometers are non-electrical devices, whereas pyrometers and resistance thermometers are electrical devices.

(a) **Mercury thermometer**

Thermal expansion and contraction of mercury (Hg) with the change in its temperature is exploited in mercury in glass or simply, mercury thermometer is used to measure temperatures of a human body, liquid and, vapour. The Mercury thermometer was first invented by Physicist Danielle Gabriel Fahrenheit in Amsterdam in 1714.

In a mercury thermometer, a glass tube is filled with mercury and the standard temperature is marked on the thermometer. These thermometers are used for clinical purposes, laboratory experiments, and industrial purposes.

Since mercury is a toxic material, mercury-free glass tube thermometers with non-toxic fluid, digital thermometers, infrared thermometers, thermocouple thermometers, liquid crystal thermometers, bimetallic thermometers, resistance thermometers, and pyrometers are also used for appropriate applications.

Bimetallic thermometer (b)

This thermometer works on the principle of thermal expansion and contraction of metals and their different temperature coefficients at the same temperature. The temperature coefficient is the relation between temperature and its effect on physical dimensions in the metal.

The bimetallic thermometer uses two connected strips of different metals e.g., steel, copper, and brass; these metal strips enlarge at different rates when they are heated which means that the difference in temperature in the two strips is converted into their mechanical displacement of a needle beside the scale. When the temperature rises, the metal strips welded with each other turn in the direction of metal with less temperature coefficient. Similarly, when the temperature reduces, the connected strips turn in the direction of metal of high-temperature coefficient.

Bimetallic thermometers are used to measure the temperature of liquids and gas. There are two types of this thermometers: (1) spiral type (2) helical type. Bimetallic thermometers are used in control devices, ovens, AC thermostats (spiral type), refineries (helical type), heaters, etc.

Fig. 6.3: Schematic diagram of a bimetallic thermometer

(c) **Platinum resistance thermometer (PRT)**

Platinum has a positive temperature coefficient, which means that it expands with an increase in temperature and that increases its resistance. It is used as a sensitive element in temperature measurement devices since it is unreactive and can be made into fine wires. In this thermometer, high purity platinum in the form of a coil or thin film is placed in a glass or metal tube and the tube is then sealed with inert gas. The contact leads connected to it through an insulating cap, are used to measure the electrical resistance of the platinum. When alternating or direct current passes through platinum its resistance changes and voltage is induced across the metal which is measured by a voltmeter. The voltmeter readings are then converted into temperatures using a calibration equation. Platinum resistance thermometers are used over a wide range of temperature (\sim -200 °C to 1000 °C) and gives precise readings. They are quite sensitive but stable over time and temperature and are easy to measure and calibrate, too. For temperature measurements above 1200 °C, platinum starts evaporating; its melting point is 1800 °C.

Fig. 6.4: Schematic diagram of platinum resistance thermometer (PRT)

(d) Pyrometer

A pyrometer is a device that measures relatively high temperatures $(> 2000 \text{ K})$ of the bodies like furnaces, molten metals, and overheated materials or liquids by measuring emitted radiations from them. This thermometer operates on Stefan's law of blackbody radiation.

In an optical pyrometer, the brightness of an object and that of filament placed inside a cylindrical pyrometer are matched for temperature measurement. Inside the cylinder, the lamp is kept between a lens and an eyepiece. The filter placed in front of the eyepiece helps to get monochromatic light. The lamp is connected with a power supply, ammeter, and rheostat. The lens focuses the radiated energy from the object and targets it onto the filament lamp. This device measures temperature with high accuracy and its accuracy depends on the filament current. An optical pyrometer is a non-contact measuring device that means direct contact with bodies or objects is not required.

A resistance pyrometer converts the change in electrical resistance caused by the heat to a temperature of an object. This instrument uses a fine wire put in contact with the object. A thermocouple pyrometer is placed in contact with the heated object and measures the output of the thermocouple which by proper calibration yields the temperature of an object.

Fig. 6.5: Block diagram of optical or radiation pyrometer

Applications (Real Life / Industrial)

Temperature measurements are essential for food industries, paper manufacturing, cold storage and pharmaceutical industries, etc.

Materials having low specific heat can be heated up or cooled down faster than those having high specific heat. This is the reason why metals whose specific heat capacity is small are used as cooking utensils. Similarly, mercury has low specific heat and therefore, it is used for temperature measurements. Water is used as a coolant in car radiators and thermal radiators due to its high specific heat capacity. An ideal coolant has a high thermal capacity.

Case-Study (Environmental/Sustainability/Social/Ethical Issues)

Since the land has a smaller heat capacity than seawater the temperature of the land increases faster than the seawater in a daytime and at night the land cools faster than seawater. During a day, hot air rises above the land due to its lower density, and cool air from the sea flows towards the land due to higher density. This is how sea breeze is produced. In the night, hot air above the sea rises and flows toward the land and cool air from the land flows towards the sea. This produces what is called land breeze.

Create Inquisitiveness and Curiosity

What types of instruments and methods are used to measure the temperature of the nuclear reactor core?

Solved Problems

Problem-1: If the specific heat of silver (Ag) is 0.235 KJ/kg·K. Then find the quantity of heat energy required to raise the temperature of 100 g of silver by 50 K.

Solution:

Specific heat of silver (Ag) = 0.235 KJ/kg.K = 0.235×10^3 J/kg.K Mass of silver = m = 100 g = 100×10^{-3} kg = 0.1 kg Temperature difference = ΔT = 50 K Therefore, the quantity of heat required = Q = m C ΔT = 0.1 × 0.235 × 10³ × 50 = 1175 J

TEMPERATURE EFFECT ON SOLID, LIQUID, AND GAS 6.2

Interesting Facts

Natural diamond has the highest thermal conductivity value (2200 WK⁻¹m⁻¹) and its value is 5 times higher than silver which has the 2nd highest thermal conductivity. But it is an electrical insulator. Due to its highest thermal conductivity, diamond is used in electronic devices for heat dispersion and can be used to check the authenticity of diamond jewellery.

$6.2.1$ **Expansion of Solids, Liquid, and Gases**

Solids have strong bonds between their molecules and when the solids are heated up at higher temperatures their molecules gain sufficient kinetic energy to vibrate with large amplitudes about their equilibrium positions. The thermal energy gained is not sufficient enough to break the bonds in solids

and make molecules free to wander in solids. As a result, solids have small expansion. In the case of liquids, bonds between molecules are weaker than those in solids. When the liquid is at sufficiently high temperature, the molecules move together and in this process, they move apart. Hence, liquids expand in volume more than solids. In gases, the molecules gain more energy at a higher temperature and they start moving faster and further apart. This causes an increase in the volume of gas. When the gas is cooled down by reducing the temperature, molecules get less energy and move slower and come closer to one another. This reduces the volume of gas at lower temperatures. Gas fills the entire space in the container in which it expands, whereas solid and liquid occupy only a certain space by expansion. Gas substances expand more than solid or liquid substances.

Coefficient of Linear, Surface, and Cubical Expansion and Relation $6.2.2$ **Amongst Them**

As the temperature of a substance changes, the change in its dimensions also occurs. If the change or an expansion is in one dimension over its volume, then it is called linear expansion. If expansion occurs in two dimensions (i.e., in surface area) of the substance, then it is called surface or superficial or arial expansion and if the expansion happens in all 3 dimensions (i.e., in volume) of the substance, then it is called volumetric or cubical expansion.

Coefficient of linear expansion: It is defined as the fractional change in the length, $\frac{dL}{L_{\infty}}$ of a substance for a change in temperature (dT) and it is denoted by α ,

i.e.,
$$
\alpha = \frac{dL}{L_0dT}
$$
 : $\alpha = \frac{dL}{dT}$ for a unit original length, L_0 of a substance.

Coefficient of surface expansion: It is defined as the fractional change in the area, $\frac{dA}{A_0}$ of a substance for a change in temperature (dT) and it is denoted by β ,

i.e., $\beta = \frac{dA}{A_0 dT}$: $\beta = \frac{dA}{dT}$ for unit original area, A_0 of a substance.

Coefficient of cubical expansion: It is defined as the fractional change in the volume, $\frac{dV}{V_0}$ of a substance for a change in temperature (dT) and it is denoted by γ ,

i.e., $\gamma = \frac{dV}{V_0 dT}$: $\gamma = \frac{dV}{dT}$ for unit original volume, V_0 of a substance.

SI unit of coefficient of thermal expansion for length, surface area, or volume is K^{-1} or ${}^{\circ}C^{-1}$.

Relation between coefficient of linear expansion and coefficient of surface expansion:

Consider a thin metal plate of length \mathcal{U}_0 , breadth \mathcal{V}_0 , and the surface area A_0 at temperature $t_1 = 0$ °C. Let the length, breadth, and area of the plate be '*f*, 'b', and 'A' respectively upon heating it at temperature $t_2 = T$ °C.

The original area of the metal plate, $A_0 = l_0 b_0$.

Note that the linear expansion is directly proportional to the original length (l_0) and the change in temperature ($\Delta T = T - 0 = T$)

i.e., Linear expansion in length, $(l-l_0) \propto l_0$ T.

Therefore, $l - l_0 = \alpha l_0$ T or $l = l_0(1 + \alpha T)$. Here, α is the coefficient of linear expansion.

Similarly, for linear expansion in breadth, we get, $b = b_0(1 + \alpha T)$.

Substituting these values in the expression of the final area, A we get,

 $A = l b = [l_0(1 + \alpha T)] \times [b_0(l + \alpha T)] = l_0 b_0 (1 + 2\alpha T + \alpha^2 T^2)$

Now, since α is very small for solids, α^2T^2 can be neglected. Hence, the above expression can be written as,

 $A = l_0 b_0 (1 + 2\alpha T) = A_0 (1 + 2\alpha T) (A_0 = l_0 b_0)$

But surface expansion can be expressed as, $A = A_0(1 + \beta T)$.

Therefore, $\beta = 2\alpha$

This gives us the relation between the coefficient of linear expansion and the coefficient of surface expansion.

Relation between coefficient of linear expansion and coefficient of cubical expansion:

Consider a thin rectangular parallelepiped of length T_0 , breadth \mathfrak{b}_0 , height \mathfrak{h}_0 , and the volume V_0 at temperature $t_1 = 0$ °C. Let the length, breadth, height, and the volume of the parallelepiped ' r ', 'b', 'h', and 'V' respectively upon heating it at temperature $t_2 = T$ °C.

Original volume of the parallelepiped, $V_0 = l_0 b_0 h_0$.

Linear expansion in length, $l = l_0(1 + \alpha T)$

Linear expansion in breadth, $b = b_0(1 + \alpha T)$

Linear expansion in height, $h = h_0(1 + \alpha T)$. Here, α is the coefficient of linear expansion.

Substituting these values in the expression of the final volume, V we get,

 $V = l b h = [l_0(1 + \alpha T)] \times [b_0(1 + \alpha T)] \times [h_0(1 + \alpha T)]$

$$
= l_0 b_0 h_0 (1 + 3 \alpha T + 3 \alpha^2 T^2 + \alpha^3 T^3)
$$

Now, since α is very small for solids, α^2T^2 and α^3T^3 can be neglected. Hence, the above expression can be written as,

 $V = l_0 b_0 h_0 (1 + 3\alpha T + 3\alpha^2 T^2 + \alpha^3 T^3) = V_0 (1 + 3 \alpha T)(V_0 = l_0 b_0 h_0)$

But cubical expansion can be expressed as, $V = V_0(1 + \gamma T)$.

Therefore, $\gamma = 3\alpha$

This gives us the relation between the coefficient of linear expansion and the coefficient of cubical expansion.

Relation between coefficient of surface expansion and coefficient of cubical expansion:

Now, since we have,

$$
\beta = 2\alpha : \alpha = \frac{\beta}{2}
$$

and $\gamma = 3\alpha : \alpha = \frac{\gamma}{3}$
Therefore, $\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$ or $6\alpha = 3\beta = 2\gamma$

6.2.3 **Coefficient of Thermal Conductivity**

Thermal conductivity is the property of a substance that measures the ability of a substance to conduct heat. It is denoted by 'K'. The inverse of the thermal conductivity is thermal resistivity.

Consider a solid material whose area of cross-section is A, is kept between two different temperatures θ_1 at $d_1 = 0$ and θ_2 at $d_2 = d$. Here, we suppose that $\theta_1 > \theta_2$. Then heat flows from higher temperature to lower temperature according to the $2nd$ law of thermodynamics. Assuming the positive direction of heat flow, the heat flux 'q' which is the heat energy per unit area per unit time can be written as

$$
q = \frac{Q}{A.t}
$$

The heat flux 'q' is directly proportional to temperature difference (θ_1 - θ_2) and inversely proportional to separation distance 'd'.

i.e.,
$$
q\alpha(\theta_1 - \theta_2)
$$
 and $q\alpha \frac{1}{d}$
Therefore, $q\alpha \frac{(\theta_1 - \theta_2)}{d}$ or $q = K \frac{(\theta_1 - \theta_2)}{d}$, where $K = \text{Coefficient of thermal conductivity.}$
We can rewrite the above expression in the form of heat energy.

$$
q = \frac{Q}{A.t} = K \frac{(\theta_1 - \theta_2)}{d} \text{ i.e., } Q = K \frac{A(\theta_1 - \theta_2)t}{d}
$$

Therefore, the coefficient of thermal conductivity, $K = \frac{Q.d}{A(\theta_1 - \theta_2)t}$. The SI unit of thermal conductivity is $WK^{-1}m^{-1}$.

Now, if A = 1 m², t = 1 sec, temperature gradient = $\frac{(\theta_1 - \theta_2)}{4}$ = 1 m⁻¹K, then K = Q.

This gives us the definition of the coefficient of thermal conductivity, K. It is defined as the heat energy flowing through a substance per its unit area per unit time per its unit temperature gradient.

$6.2.4$ **Engineering Applications**

Materials with high thermal conductivity are good thermal conductors and they transfer heat effectively. They can be used as a heat sink in electronics and appliances. Metals like Al, Cu, Ag, Au, Be and graphite, aluminum nitride, silicon carbide, etc. are good thermal conductors.

Because silver is an efficient thermal conductor its paste is used in the production of photovoltaic cells and copper is used for making utensils, hot water pipes, and electronic heat sinks due to its high thermal conductivity. Aluminum foils are used to keep food warm and are also used to make cooking pans. The ability to conduct heat effectively makes iron very useful in vehicle engine manufacturing and in car radiators. Materials with low thermal conductivity are poor heat conductors and do not transfer heat readily. They can be used as thermal insulators or heat resistants, e.g., rubber, foam, plastics, glass, mica, quartz, etc. are poor conductors of heat. Thermal insulators are used for protection against undesirable excess heat.

Applications (Real Life / Industrial)

The concept of thermal expansion and contractions is used in mercury thermometers, and bimetallic thermometers, to join steel plates using rivets, for opening the tight lid of a jar by passing hot water, for firing and fixing iron rims on wooden wheels of a cart, in thermostats, in engine coolant to avoid overflowing, in setting up the powerlines considering its sagging due to thermal expansions and in laying the railway tracks or constructing bridges with a small gap or expansion joints to avoid buckling due to thermal expansion in hot summer days. Thermal expansion of the gases or air is taken into consideration while inflating tyres in summer.

Case-Study (Environmental/Sustainability/Social/Ethical Issues)

Water shows unusual property called anomalous thermal expansion when it is cooled below 4 C° until it reaches $0^{\circ}C$. If it is cooled further, its volume increases suddenly while changing to the solid phase i.e., ice at 0°C. When ice is cooled below 0°C, its volume shrinks like in solids due to thermal contraction. Due to this anomalous thermal expansion of water, marine life is protected in the Arctic Ocean during extremely cold days.

Create Inquisitiveness and Curiosity

How electrical conductivity and thermal conductivity of metals are related to each other?

Solved Problems

Problem-1: Two metals, P and Q, have the same sizes. The thermal conductivity of the metal $P = 2k$ and the thermal conductivity of the metal $Q = k$. The end of both metals joined together has different temperatures, $T_{1(P)} = 200\text{°C}$ and $T_{2(O)} = 20\text{°C}$. What is the temperature at the interface of both metals? Solution:

Since both metals have the same sizes, separation distance 'd' from the interface and the area of crosssection 'A' are the same for both P and Q. Let the temperature of the interface be $T^{\circ}C$.

Here, $K_p = 2k$ and $K_Q = k$; $T_{1(P)} = 200$ °C and $T_{2(Q)} = 20$ °C

Then, rates of heat transfer through conduction in metal P and Q are given by

$$
\left(\frac{Q}{t}\right)_P = \frac{K(T_1 - T)A}{d} = \frac{2k(200 - T)A}{d}
$$

And

$$
\left(\frac{Q}{t}\right)_Q = \frac{K(T - T_2)A}{d} = \frac{k(T - 20)A}{d}
$$

At the interface of P and Q, we get

$$
\left(\frac{Q}{t}\right)_P = \left(\frac{Q}{t}\right)_Q
$$

$$
\therefore \frac{2k(200 - T)A}{d} = \frac{k(T - 20)A}{d}
$$

$$
\therefore 2k(200 - T) = k(T - 20)
$$

$$
\therefore 400 - 2T = T - 20
$$

$$
\therefore 420 = 3T
$$

$$
\therefore T = 140^{\circ}C
$$

UNIT SUMMARY

Heat is thermal energy and it is an ability to perform work, temperature is a measure of the \bullet intensity of the heat.

 $[LOD2]$

 $[LOD3]$

- Specific heats are calculated for a unit mass of a substance in kilogram, gram or pounds whereas \bullet molar heat capacity is calculated for a unit mole of a substance.
- Solids, liquids, and gases all expand when they are heated and they all contract when cooled \bullet down. The expansion or contraction occurs due to increased or decreased movement of their molecules upon heating or cooling.
- Thermal conductivity is the property of a material to conduct heat. Some materials are good thermal conductors like metals and some are poor conductors of heat or thermal insulators like rubber.

EXERCISES

(A) Objective Questions

- (B) anomalous expansion
- (C) thermal expansion
- (D) thermal conductivity

(A) **Answers of Objective questions**

- A6.1 calorimeter
- A6.2 γ
- A6.3 joule
- $A6.4$ (A) True
- A6.5 (C) thermal expansion

(B) **Subjective Questions**

- 6.1 Give the difference between heat and temperature. [LOD1] 6.2 The boiling point and freezing point of Acetone are 56.2 °C and - 94.8 °C. Convert these values
- into Fahrenheit, Kelvin, and Rankine scale.
- 6.3 Explain the mode of heat transfer in liquid metals.
- 6.4 Good bricks are the ones that have low thermal conductivity to keep the house cool in summer and warm in winter. Red bricks have thermal conductivity = 0.6 WK⁻¹ m⁻¹. If the temperature at the one end of such brick is 45 °C and at the other end it is 26 °C. The separation distance between the two ends is 10 cm. Find the time rate of heat flux passing through the brick. $[LOD2]$
- 6.5 Differentiate between thermal conductors and thermal insulators. $[LOD2]$

Answer of Subjective questions (B)

A6.2. Boling point of Acetone: 56.2 °C = 133.16 °F = 329.3 K = 592.74 °R Freezing point of Acetone: -94.8 °C = -138.64 °F = 178.3 K = 320.94 °R [Hint: Use the formula for temperature scale conversion]

A6.4. 114 watt m⁻² [Hint: Use this formula for rate of heat flux: $\frac{Q}{A_f} = \frac{K(T_1 - T_2)}{A}$ and use appropriate conversion of units.]

PRACTICALS

To find the coefficient of linear expansion of the material of a rod. $\mathbf 1$

Practical significance

Materials especially, metals expands when their temperature is raised because of an increase in an average separation between their atoms. The expansion in length of a material is determined using linear thermal expansion coefficient. For metals, the coefficients of thermal expansion are of the orders of 10^{-5} per °C. This can be determined using micrometer screw linear thermal expansion apparatus which requires the material in the rod shape.

Relevant theory

For theory, refer to Unit 6 (Section: 6.2.1 and 6.2.2)

Formula: Linear thermal expansion coefficient,

 $\alpha = \frac{1}{L_0} \frac{dL}{dT}$ or $\frac{\Delta L}{L_0 \Delta T} = \frac{\Delta L}{L_0 (T_f - T_{\text{Room}})} (^{\circ}C)^{-1}$ Where, L_0 = Original length of the rod; dL or ΔL = extension in the length of rod; dT or ΔT = difference in temperature; T_f = Final steady temperature (°C); T_{Room} = Room temperature (°C)

Practical outcomes (PrO)

The practical outcomes are derived from the curriculum of this course:

- PrO1: Acquire skills in setting up the experiment and precise measurement of temperature using thermometer.
- PrO2: Describe thermal expansion in solids and determine the linear thermal expansion of a rod of different metals.
- PrO3: Operate and control the required equipment with proper precautions either in a group or individually.

Practical setup (drawing/sketch/circuit diagram/work situation)

Figure on the left: linear expansion apparatus. Here, the rods are placed inside the silver jacket and the whole assembly is placed on the black base. (Photo credit: IISER Pune, (http://www.iiserpune.ac.in/ ~bhasbapat/phy221_files/lab1.pdf) (As on 31st July 2021))

Fig 6.6: Experimental setup for determining the linear thermal expansion of a metal rod

Resources required

Precautions

- 1. Be extra careful while handling the steam generator or boiler, rubber tubes, hot plates and linear expansion apparatus. The steam due to its higher heat capacity can be hazardous and may cause severe burns if you remain inattentive.
- $2.$ Keep your face away from the opening of steam outlets.
- $3.$ Stoppers and caps should be loosely inserted in order to prevent building up the pressure and subsequent steam explosion or release of high-pressure steam jet.

Make sure to measure and record the room temperature and the length of each metal rod before 4. heating them for measuring their expansion.

Suggested procedure

- Fill the steam generator or boiler with water up to its two-thirds level and then turn it on. 1.
- $\overline{2}$. Insert a rod into the jacket and place it on the base. Adjust the micrometer screw such that it touches the rod and the other end of the rod should touch the fixed screw. Use micrometer screw gauge to read the length of the rod.
- $3.$ Note the length of the rod for given materials and the room temperature before heating up any of the rods.
- $4.$ Connect the tubing from the steam generator or boiler to the linear expansion apparatus. The tubing from linear expansion apparatus into a beaker should be well below the level of the apparatus.
- 5. Place the thermometer properly in the opening provided in the expansion apparatus. Allow steam to flow through the jacket until a steady temperature is reached.
- Then, turn the micrometer screw until it is properly fit and record the readings of the 6. micrometer screw as the difference in the length of rod, and the thermometer. Turn off the water and drain the jacket. Turn back the micrometer screw.
- 7. Replace the rod with that made from different metal and repeat steps 4 through 6 for this new rod.
- 8. Disconnect the apparatus, empty the steam generator or boiler and beaker, mop up the spilt water and leave the working area clean to prevent any accidents.

Observations and calculations

Observations

Room temperature $(T_{\text{Room}}) =$ __________ °C

Observation table

Calculation

From theory: (calculate for linear thermal expansion for each rod of different material) Linear thermal expansion in a rod,

$$
\alpha = \frac{\Delta L}{L \cdot \Delta T} = \frac{\Delta L}{L \cdot (T_f - T_{Room})} ({}^oC)^{-1}
$$

% Error in linear thermal expansion

 $\frac{\alpha_{\text{standard}} - \alpha_{\text{experimental}}}{\alpha_{\text{standard}}}$ × 100

Results and/or interpretation

- The linear thermal expansion coefficient of an aluminum rod is $___$ ($^{\circ}$ C)⁻¹. $1.$
- $2.$ The linear thermal expansion coefficient of an iron/brass/copper rod is ρ (°C)⁻¹.
- $3₁$ % Error in linear thermal expansion coefficient of an aluminum rod = $_____\$
- % Error in linear thermal expansion coefficient of an iron rod = $____\$ % $\overline{4}$.

Conclusions and/or validation (To be filled by student)

Practical related questions (Use separate sheet for answer)

- How can you interpret the linear expansion coefficients for aluminum and iron? 1.
- $2.$ What is the principle of a mercury thermometer?
- $3₁$ Can you approximate the coefficient of volume expansion for your results? Give the answers in terms of α .
- $4.$ List out the real-life applications or usages of thermal expansion in solids.
- A copper wire of length 0.8350 m at 34.7 °C is left outside overnight in the extreme cold 5. temperature. Assuming that the night temperature reaches the freezing temperature, calculate the new length of the copper wire.

Disposal of waste

Classify the waste materials to be thrown in this experiment in the following bins:

Environment friendly approach: reuse, reduce and recycle

Required apparatus for this practical are reusables for very long time if they are operated carefully.

Suggested assessment scheme (To be filled by teacher)

The given performance indicators should serve as a guideline for assessment regarding process and product related marks.

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* Marks and percentage weightages for product and process assessment will be decided by the teacher.

2. To measure room temperature and temperature of a hot bath using mercury thermometer and convert it into different scales.

Practical significance

Temperature measurement with accuracy is very crucial in science and industry, especially in various industrial processes, in internal combustion engines and in numerous physical phenomena. Science deals with Kelvin scale of temperature, while in daily life, degree Celsius scale of temperature is used. For a human body, Fahrenheit scale of temperature is generally used. Hence, it is essential to know the unit conversion factors.

Relevant theory

For theory, refer to Unit 6 (Section: 6.1.4 and 6.1.5(a))

Practical outcomes (PrO)

The practical outcomes are derived from the curriculum of this course:

- PrO1: Acquire skills in accurate measurement of temperature of various objects at different temperatures and convert them in different temperature scales with accuracy.
- PrO2: Describe the use of thermometers to measure temperature under different conditions and different scales of temperature measurements.
- PrO3: Operate the mercury thermometer with care and proper precautions considering extreme toxicity of mercury.
Practical setup (drawing/sketch/circuit diagram/work situation)

Figure Refer to Unit 6 (Section: 6.1.4) figure 2 for temperature conversion.

Resources required

Precautions

- 1 Due care must be taken while dealing with warm to hot water or castor-oil.
- Thermometer contains mercury which is extremely toxic in nature, so it must be handled with care. $2.$
- Mercury bulb should not be touched before placing it into liquids. Hold the thermometer from $3.$ the top.

Suggested procedure

- $\mathbf{1}$. Shake the thermometer gently before measuring the room temperature.
- $2.$ Either hang or hold the thermometer from the top in a ventilated and spacious room for a minute and note the reading of mercury level in the graded thermometer.
- $3.$ Now, place a beaker or container on a flat and hard surface.
- Pour hot water in it and place a thermometer in the container such that mercury bulb comes in $4.$ direct contact with the water. Let the thermometer be in this state for one minute and measure the temperature.
- Similarly, repeat the experiment for warm to hot castor-oil. 5.

Observations and calculations

Observations

- Range of mercury thermometer = $\rule{1em}{0.15mm}$ to $\rule{1em}{0.15mm}$ $^{\circ}$ C $1.$
- Least count of mercury thermometer = $__$ °C $2.$
- $3.$

Observation Table

Calculation

Celsius scale to Fahrenheit scale: °F = $\frac{9}{5}$ °C+32 Celsius scale to Kelvin scale: $K = 273.15 + C$ Kelvin scale to Rankine scale: ${}^{\circ}R = \frac{9}{5}K$

Results and/or interpretation

Conclusions and/or validation (To be filled by student)

Practical related questions (Use separate sheet for answer)

- At what temperature does the water boil in Kelvin and Fahrenheit scales? 1.
- $2.$ What is a triple point?
- $3.$ Convert the value of a triple point of H2O in Fahrenheit scale.
- 4. Why should you not touch the bulb of mercury before measuring temperature?
- $5.$ Does the temperature of a human body remain same for all age group? Why?

Disposal of waste

Classify the waste materials to be thrown in this experiment in the following bins:

Environment friendly approach: reuse, reduce and recycle

Castor-oil and thermometer both are reusable for long time. For storing castor-oil, keep it away from sunlight and humidity. Water after it cools down, can be used to water nearby plants.

Suggested assessment scheme (To be filled by teacher)

The given performance indicators should serve as a guideline for assessment regarding process and product related marks.

* Marks and percentage weightages for product and process assessment will be decided by the teacher.

KNOW MORE

- Kelvin scale uses the same size degree (i.e., 100 degrees) as Celsius scale and Rankine scale uses the same size degree (180 degrees) as Fahrenheit scale.
- A bolometer is a device that measures electromagnetic radiation and heat. It uses a temperaturesensitive resistive element. Bolometers measure radiations and heat from particle detectors and are used for thermal cameras, fingerprint scanners; detection of forest fire, concealed weapons, and astronomical radiations. Bolometers have promising applications in thermal imaging, THz communication, and solar probes. Pyrometers are akin to bolometers and thermistors for their use in thermometry.

Innovative Practicals /Projects/ Activities

- Prepare a chart or PowerPoint presentation about the principle, working, and applications of (1) thermistors.
- Make a list of different methods to measure the thermal conductivity of solids, liquids, and gases (2) and prepare a presentation individually or in a group.
- (3) Discuss in the group and list out major causes of industrial fire and explosion and measures of prevention. Make a presentation about industrial fire safety.

REFERENCES AND SUGGESTED READINGS

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Suggested Learning Resources for Practical

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ANNEXURES

Annexure I: Important Formulae

Unit 1: Physical world, Units and Measurements

Formula for conversion of value of given physical quantity from one system of unit to another $\mathbf{1}$. $n_1u_1 = n_2u_2$

 n_1 = numerical value in first system of unit, u_1 = unit of physical quantity in first system of unit, n_2 = numerical value in second system of unit, u_2 = unit of physical quantity in second system of unit

 $\overline{2}$. The least Count (L.C) of Vernier Calipers is given by the following formula:

Least Count = $\frac{\text{smallest division on main scale}}{\text{Total number of divisions on Vernier scale}} = \frac{\text{SDMS}}{\text{TDVS}} = \frac{1\text{MSD}}{\text{n}}$

Least count of micrometer screw gauge = $\frac{\Delta W}{\Delta W}$ of divisions on circular scale

- Least Count of spherometer = $\frac{\text{Pitch of the spherometer screw}}{\text{Number of divisions on circular scale}}$ 4.
-
- 5. If values obtained are a1, a2, a3,an. then the arithmetic mean of these values is given as:

$$
\overline{a} = a_{\text{mean}} = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}
$$

 $3.$

- 6. Then the errors in the individual measurement are written as, $\Delta a_1 = \overline{a} - a_1$; $\Delta a_2 = \overline{a} - a_2$; $\Delta a_3 = \overline{a} - a_3$, $\Delta a_n = \overline{a} - a_n$
- **Mean Absolute Error** $(\overline{|\Delta a|})$: It is represented by $\Delta \overline{a}$ or Δa_{mean} . 7.

Thus,
$$
|\overline{\Delta a}| = \overline{\Delta a}_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \dots + |\Delta a_n|}{n}
$$

8. Relative error
$$
(\delta a) = \frac{|\Delta a|}{\overline{a}} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}
$$
 and

9. Percentage error,
$$
\delta a = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%
$$

- 10. Errors in a summation $(Z = X + Y)$ or a subtraction $(Z = X - Y)$: The maximum possible errorin Z for either summation or subtraction is given by, $\Delta Z = \Delta X + \Delta Y$
- Errors in a multiplication ($Z = XY$) and a division ($Z = X/Y$): The maximum relative or fractional 11. error in either multiplication or division is given by $\frac{\Delta Z}{Z} = \frac{\Delta X}{X} + \frac{\Delta Y}{V}$.

12. Error in a quantity with powers: Suppose,
$$
Z = k \frac{X^n Y^m}{C^q}
$$
 where, $k = \text{constant}$. Then the relative
or fractional error can be written as $\frac{\Delta Z}{Z} = n \frac{\Delta X}{X} + m \frac{\Delta Y}{Y} + q \frac{\Delta C}{C}$.

Unit 2: Force and Motion

- Unit vector of vector is denoted as $\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\text{vector}}{\text{magnitude of vector}}$ $1.$
- The magnitude of $(\vec{a} + \vec{b})$ is given as $|\vec{a} + \vec{b}| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$ where θ is a smaller $2.$ anglebetween the vector \vec{a} and \vec{b} .
- If $(\vec{a} + \vec{b})$ is making an angle α with vector \vec{a} then, $\alpha = \frac{bsin\theta}{a + b\cos\theta}$ 3.
- If vector \vec{A} makes an angle α with X-axis and $\beta = (90 \alpha)$ with Y-axis, then rectangular $\overline{4}$. component along X- axis, $A_x = A \cos \alpha$ and component along Y-axis, $A_y = A \cos \beta$, thus vector $\vec{A} = A_x \vec{i} + A_y \vec{j} = (A \cos \alpha) \vec{i} + (A \cos \beta) \vec{j}$

If components Ax and Ay are given then the magnitude of vector $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$ and 5.

- $\tan \alpha = \frac{A_y}{A_x}$
Let $\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$, $\vec{B} = B_x \vec{i} + B_y \vec{j} + B_z \vec{k}$ 6. $\vec{A} \pm \vec{B} = (A_x \pm B_x) \vec{i} + (A_y \pm B_y) \vec{j} + (A_z \pm B_z) \vec{k}$ Magnitude $|\vec{A} \pm \vec{B}| = \sqrt{(A_x \pm B_x)^2 + (A_y \pm B_y)^2 + (A_z \pm B_z)^2}$
- The dot product of two vectors \vec{A} and \vec{B} is given as 7. $\vec{A} \cdot \vec{B} = AB \cos \theta = |\vec{A}| |\vec{B}| \cos \theta$ where θ is angle between vector \vec{A} and \vec{B} $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$
- Vector product or cross product of two vectors \vec{A} and \vec{B} is denoted by $\vec{A} \times \vec{B}$ and it is also a 8. vector itself. Then magnitude of this vector given as $|\vec{A} \times \vec{B}| = AB \sin \theta$

$$
\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \vec{i} + (A_z B_x - A_x B_z) \vec{j} + (A_x B_y - A_y B_x) \vec{k}
$$

- External applied force $\vec{F} = k \frac{m(\vec{v}_2 \vec{v}_1)}{\Delta t}$ or $\vec{F} = m\vec{a}$ 9.
- 10. The thrust of rocket = mass x acceleration of rocket = velocity of gases x rate of mass decrease = $\vec{F} = m \frac{(\Delta \vec{v})}{\Delta t} = \frac{\Delta m}{\Delta t} \vec{u}$
- Impulse $\vec{l} = \vec{F}\Delta t = \vec{p}_2 \vec{p}_1$ 11.
- Instantaneous angular velocity $\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$ 12.
- Instantaneous angular acceleration $\alpha = \lim_{n \to \infty} \frac{\Delta \omega}{\Delta n} = \frac{d\omega}{\Delta n}$ 13.
- Relation between linear velocity and angular velocity $v = r\omega$ 14.
- 15. Relation between linear acceleration and angular acceleration $a = r\alpha$
- 16. The equations for constant angular acceleration are given below

(1)
$$
\omega = \omega_0 + \alpha t
$$
 (2) $\theta = \omega_0 + \frac{1}{2}\alpha t^2$ (3) $\omega^2 = \omega_0^2 + 2\alpha \theta$

- Centripetal force F_c = mass × centripetal acceleration = mv²/r = m ω ²r 17.
- Velocity of vehicle on banked road $v = \sqrt{rg \tan \theta}$ 18.

Unit 3: Work, Power and Energy

- Work (W) = $\mathbf{F} \cdot \mathbf{r} = \mathbf{F} \cos(\theta)$, r, for variable force work $W = [F dr,$ 1.
- Limiting Friction = $\mu_{\text{limiting}} \times$ (Normal Force (or normal reaction force) = μ_{limiting} N $\overline{2}$.
- Limiting static friction, $f_s = \mu_s N = \mu_s mg$ and kinetic friction force $f_k = \mu_k N$ 3.
- On inclined plane $\mu_k = \mu_s = \tan \theta$ 4.

5. kinetic energy (KE) =
$$
\frac{1}{2}
$$
mv² and gravitational potential energy U = mgh

6. Power
$$
P = \frac{dW}{dt} = \frac{F \cdot dr}{dt} = F \cdot v
$$
 $(\because W = F \cdot r \text{ and } \frac{dr}{dt} = v)$

Unit 4: Rotational Motion

- Torque $\vec{\tau} = \vec{r} \times \vec{F} = F \sin \theta$ 1.
- Angular momentum $\vec{L} = \vec{r} \times \vec{p} = mvr \sin \theta$ $\overline{2}$.
- Moment of inertia I = $m_1r_1^2 + m_2r_2^2 + \cdots + m_nr_n^2 = \sum_{i=1}^n m_i r_i^2$ 3.
- Relation between torque and moment of inertia $\tau_{\text{net}} = \sum_{i=1}^{n} m_i r_i^2 \alpha = I \alpha$ 4.
- Relation between angular momentum and moment of inertia $L_{net} = \sum_{i=1}^{n} m_i r_i^2 \omega = I \omega$ 5.
- Radius of gyration $K = \sqrt{\frac{\sum_{i=1}^{n} m_i r_i^2}{M}}$ where moment of inertia of body I = M \times K² 6.

7. Perpendicular axes theorem
$$
I_z = I_x + I
$$

- Parallel axes theorem $I_{AB} = I_C + Md^2$ 8.
- When torque = 0 then $I_1 \omega_1 = I_2 \omega_2$ where $\frac{dI}{dt} = \vec{\tau}_{net}$ 9.
- M.I. of rod about an axis of rotation passes through the Centre and perpendicular to the length I 10. $=\frac{MI^{2}}{12}$

M.I. of disc about axis = $\frac{1}{2}$ Mr², about diameter = $\frac{1}{4}$ Mr², about tangent to rim, parallel to axis = 11. $\frac{3}{2}$ Mr², about tangent to rim, parallel to diameter = $\frac{5}{4}$ Mr²

M.I. of ring about axis I = Mr², about diameter I = $\frac{1}{2}$ Mr², about tangent to rim, parallel to axis = 12. 2Mr², about tangent to rim, parallel to diameter = $\frac{3}{2}$ Mr²

M.I. of solid sphere about diameter I = $\frac{2}{5}$ Mr², about tangent = $\frac{7}{5}$ Mr² 13. M.I. of hollow sphere about diameter $I = \frac{2}{5}M\left(\frac{r_2^5 - r_1^5}{r_2^3 - r_1^3}\right)$ 14. M.I. of spherical shell about diameter I= $\frac{2}{3}$ Mr² 15.

Unit 5: Properties of Matter

1. Young s modulus Y =
$$
\frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{F_A}{\Delta L} = \frac{FL}{A \Delta L}
$$

\n2. Bulk modulus B = $\frac{\text{Volumetric stress}}{\text{Volumetric strain}} = \frac{\Delta P}{-\Delta V_V} = \frac{-V(\Delta P)}{\Delta V}$, compressibility K = 1/B
\n3. Modulus of rigidity $\eta = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F_A}{\chi} = \frac{FL}{Ax}$
\n4. Pressure P = $\frac{F}{A}$ and absolute pressure P = P_{atm} + pgh, gauge pressure = P - P_{atm} = pgh
\n5. Surface tension S = $\frac{F}{1}$ and ascent formula in capillary h = $\frac{2\text{scos}\theta}{rpg}$
\n6. Viscous force F = $\eta A \frac{(v_2 - v_1)}{d}$, where $\frac{(v_2 - v_1)}{d}$ is a velocity gradient
\n7. Relation between kinetic viscosity and dynamic viscosity $\mu = \frac{\eta}{\rho}$
\n8. Terminal velocity V_t = $\frac{2r^2g}{9}(\rho - \rho_f)$
\n9. Stokes' law F_V = $6\pi\eta rV$
\n10. Temperature dependence of viscosity in liquids $\eta = \frac{\eta_0}{(1 + \alpha t + \beta t^2)}$
\n11. Temperature dependence of viscosity in gases $\eta = \eta_0 + \alpha t + \beta t^2$
\n12. Reynolds number, $(R_e) = \frac{\rho V D}{\eta} = \frac{VD}{\mu}$
\n13. Continuity equation A₁V₁ = A₂V₂, and AV = constant = R = volume flow rate
\n14. Bernoulli's equation P + pgh + $\frac{1}{2}\rho V^2$ = Constant

Unit 6: Heat and Thermometry

- Heat supplied to (or taken out of) a substance, $Q = C \Delta T = C (T_2 T_1)$ $1.$
- Heat supplied, Q = n C_m ΔT = n C_m (T₂ T₁), where C_m = molar heat capacity $2.$ \overline{C}

3.
$$
\gamma = \frac{P}{C_v} \text{ and } C_p - C_V = R
$$

4.
$$
^{\circ}F = \frac{9}{5}^{\circ}C + 32
$$
, $K = 273.15 + ^{\circ}C$, $^{\circ}R = ^{\circ}F + 459.67$, $^{\circ}R = \frac{9}{5}K$
dl. dA dV

5.
$$
\alpha = \frac{dL}{L_0 dT}
$$
, $\beta = \frac{dA}{A_0 dT}$ and $\gamma = \frac{dV}{V_0 dT}$

6.
$$
I = I_0(1 + \alpha T), A = A_0(1 + \beta T) \text{ and } V = V_0(1 + \gamma T)
$$

$$
7. \qquad \alpha = \frac{\beta}{2} = \frac{\gamma}{3}
$$

8. Heat energy Q =
$$
K \frac{A(\theta_1 - \theta_2)t}{d}
$$
, where temperature gradient = $\frac{(\theta_1 - \theta_2)}{d}$

Annexure II: CONVERSION FACTORS

Length

 $1 m = 100 cm = 3.28 ft = 39.37 in.$ $1 \text{ A}^{\circ} = 10^{-10} \text{ m} = 10^{-8} \text{ cm}$ 1 light year = 9.46×10^{15} m $1 \text{ mi} = 1.609 \text{ km}$ 1 ft = 0.3048 m = 30.48 cm 1 inch (in.) = 2.54 cm **Mass** $1 \text{ kg} = 10^3 \text{ g}$ 1 Pound = 0.454 kg $1 u = 1$ amu = 1.66 $\times 10^{-27}$ kg **Time** = 1.013×10^5 Pa 1 min. $= 60 s$ $1 h = 60 min. = 3600 s$ 1 day = $86400 s$ Volume 1 litre = $1000 \text{cm}^3 = 10^{-3} \text{ m}^3$ Angle 1] = 0.239 cal. 1 rad = $180^\circ/\pi = 57.3^\circ$

 $1^\circ = \pi/180$ rad = 1.745 \times 10⁻² rad

1 revolution = 2π rad = 360°

Acceleration 1 St = $1 \text{ cm}^2\text{/s}$

 $1 \text{ ft/s}^2 = 0.3048 \text{ m/s}^2 = 30.48 \text{ cm/s}^2$

Speed = 252 cal. $1 \text{ km/h} = 0.2778 \text{ m/s}$ $1 \text{ mi/h} = 0.447 \text{ m/s}$

Force $1 N = 10^5$ dyne $11b = 4.448 N$ Power $1 W = 1 I/s$ $1 \text{ kW} = 10^3 \text{ W}$ 1 hp = $746 W$ Pressure $1 Pa = 1 N/m²$ 1 bar = 10^5 Pa 1 atm. = 1.013 bar $1 \text{ torr} = 1 \text{mm Hg}$ $= 133.3$ Pa Energy 1 cal. = 4.186 J $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ $1I = 10⁷$ ergs 1 kWh = 3.6×10^6 J 1 Btu = 1055 J **Viscosity** 1 Pa-s = 1 Pl = 10 P

Annexure III: Some general and specific instructionswhen working in the laboratory

General Instructions

- 1. In the laboratory, work quietly and cautiously. Remember the main purpose of doing any experiment is to make faithfulmeasurements.
- 2. Always share equallyall the steps of the work with your partner.
- 3. Presentationsofdata intabular form, graphs and calculations should be done correctly and sincerely.
- 4. Be always honest at the time of recording and representing the experimental data. Avoid manipulating the readings.
- 5. If any reading appears incorrect, you have to repeat the measurementagain and again to find the source of error.
- 6. Plot the graphs properly using carefully obtained data from the experiments.
- 7. You get exposed to measuring instruments and devices for the sole purpose of being familiar with them and developing the motor skills eventually so grab the opportunity to focus on the fine points and learn them.
- 8. Cultivate and grow an attitude of learning through performing experiments with interest to verify the theoretical knowledge you have gathered.
- 9. Punctuality is the key to success. Always beon time for the laboratory sessions and come prepared with a clear knowledge abouttheexperiment.

Specific Instructions

- 1. When working in the laboratory for collecting data of your experiment, it is important to note all the measured data neatly in the notebook.
- The recorded data entered in the notebook have to confirm by your instructor before leaving the $2.$ laboratory.
- 3. All the students doing the same experiment have to maintain individual copy of the recorded data. The laboratory notebook is required to bring in the laboratory regularly when you come for doing the experiment.
- 4. Graphs are to be drawn properly at the end of each of experiment.
- 5. For this you need to know how to optimize on usage of graph paper. Remember all the repeated data are to be accommodated on a single graph sheet.
- 6. Graphs are to be labeled properly along with the axes showing the corresponding units.
- 7. During the working hours in the laboratory, you are supposed to fully utilize the duration and do not leave the laboratory before the completion of the working hours. If you finish early, you may spend the remaining time to complete the calculations and graphs drawing and for that in the laboratory you are supposed to come equipped with calculators, pencils and scale.

APPENDICES

APPENDIX A: Suggestive Template for Practical

Aim

Explain briefly about the aim of the experiment.

Relevance

Explain about the relevancy of the experiment in your own words.

Requirements

List out all the required apparatus along with their proper specifications.

Procedure, Observations and Inference

Explain the procedure of the experiment step-wise and note the observations properly. On the basis of observations certain inference is to be made. You can use a table similar to that given below

Video / animation

If possible, you can go through some video/animation to visualize the steps physically.

Calculations

Properly calculate all the required physical quantities essential for your experiment.

Result and Discussion (Error measurement)

Obtain the final result and discuss about it with proper considerations of errors which can be introduced during your experiment.

Conclusions

Finally give your conclusion based on the obtained results.

Validation of the topics in Experiment

Try to validate the result of the experiment in real life scenario.

Use of ICT

You can also study using the available online resources. These are useful as there is no time constraint at all. Some of which are listed (not limited to) below:

https://swayam.gov.in/ https://nptel.ac.in/ https://www.swayamprabha.gov.in/

Note for Instructor and Lab-Technicians

Some general and specific instructions are listed separately [see Annexure III] for laboratory preparation, maintenance, safety aspects, etc. Laboratory Instructor and Lab-Technicians can follow those instructions properly to run the laboratory smoothly without any hazard.

APPENDIX B: Indicative Evaluation Guidelines for Practical / Projects / Activities in Group

Process Related Skills

Product Related Skills

APPENDIX C: Assessments Aligned to Bloom's Level

- **Bloom's Taxonomy–** It has been coupled into following two categories for development of Questions for this Quadrant as given below:

APPENDIX D: Records for Practical

REFERENCES FOR FURTHER LEARNING

Lists of some of the books are given below which may be used for further learning of the subject (both theory and practical) by the interested students:

- $\mathbf{1}$. Serway, Raymond A.; Jewett, John W., Physics for Scientists and Engineers with Modern Physics (9th ed.), Cengage Learning, 2017.
- $\overline{2}$. Tipler, Paul, Physics for Scientists and Engineers: Vol. 1 (4th ed.), W. H. Freeman, 1998.
- $3.$ Walker, Jearl, The Flying Circus of Physics (2nd ed.), Wiley & Sons, 2006.
- Bloomfield, Louis A., How Things Work: The Physics of Everyday Life (6th ed.), Wiley & Sons, 2019. 4.
- 5. Griffith, W. Thomas; Brosing, Juliet, The physics of everyday phenomena: A conceptual introduction to physics (9th ed.), McGraw Hill, 2019.
- 6. Young, Hugh; Freedman, Roger, University Physics with Modern Physics (14th ed.), Pearson, 2017.
- $7.$ Lamb, H., Hydrodynamics (6th ed.), Cambridge University Press, 1994.
- 8. Giancoli, Douglas, *Physics for scientists and engineers* (5th ed.), Pearson, 2021
- 9. Hewitt, Paul G, Conceptual Physics (12th ed.), Pearson, 2017
- 10. Moore, Thomas, Six Ideas that Shaped Physics: Unit T Some Processes are Irreversible (3rd ed.) McGraw-Hill Education, 2016
- 11. Halliday, David, Resnick, Robert and Walker, Jearl, Fundamentals of Physics (11th ed.) Wiley & Sons, 2018

List of open education resources

- (1) http://www.sciencefairadventure.com/
- (2) http://www.physicsclassroom.com/
- (3) http://www.physics.org/
- (4) http://www.fearofphysics.com/
- (5) http://www.sciencejoywagon.com/physicszone/
- (6) http://www.science.howstuffworks.com/
- (7) https://swayam.gov.in/
- (8) https://nptel.ac.in/
- (9) https://www.swayamprabha.gov.in/

CO AND PO ATTAINMENT TABLE

Course outcomes (COs) for this course can be mapped with the programme outcomes (POs) after the completion of the course and a correlation can be made for the attainment of POs to analyze the gap. After proper analysis of the gap in the attainment of POs necessary measures can be taken to overcome the gaps.

Table for CO and PO attainment

The data filled in the above table can be used for gap analysis.

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